14. SUPPLEMENT

Some mathematical deductions

Chapter 2-*page 10 (f_v)*

The mean force of all *pp*'s working over the period Δt can be found conform:

$$f_v = \int_{-\Delta t/2}^{\Delta t/2} \frac{(\delta e)^2 .dt}{(A - v.t)^2} = \frac{(\delta e)^2}{\Delta t.v} \cdot \left[\frac{1}{A - v.t} \right]_{-\Delta t/2}^{\Delta t/2} = \frac{(\delta e)^2}{\Delta t.v} \cdot \left[\frac{1}{A - \Delta t.v/2} - \frac{1}{A + \Delta t.v/2} \right] =$$

$$= \frac{(\delta e)^2}{\Delta t.v} \cdot \left[\frac{A + \Delta t.v/2 - A + \Delta t.v/2}{A^2 - (\Delta t.v/2)^2} \right] = \frac{(\delta e)^2}{A^2 - (\Delta t.v/2)^2}$$

The average force exerted on δe over the period Δt is presented by:

$$f_v = f_0 \cdot \frac{1}{1 - (\Delta t.v/2A)^2} \text{, where } f_0 = \frac{(\delta e)^2}{A^2}$$

Chapter 3-*pages 16/17 (f_1 and f_2)*

$$f_1 = \frac{k.e_1.e_2.D.(A + x)}{\left\{(A - x)^2 + y^2\right\}^{3/2}} = \frac{k.e_1.e_2.D.(A + x)}{\left\{A \;\; + 2A.x + x^2 + y^2\right\}^{3/2}}$$; x = -v.cosα.t ; y = +v.sinα.t ; k = $1/(4\pi\varepsilon_0)$

$$f_1 = \frac{k.e_1.e_2.D.(A - v.t.\cos\alpha)}{\left(A^2 - 2A.v.t.\cos\alpha + v^2.t^2.\cos^2\alpha + v^2.t^2.\sin^2\alpha\right)^{3/2}} = \frac{k.e_1.e_2.D.(A - v.t.\cos\alpha)}{\left(A^2 - 2A.v.t.\cos\alpha + v^2.t^2\right)^{3/2}}$$

The momentum of f_1 is presented by:

$$\int f_1.dt = k.e_1.e_2.D.A.\int \frac{dt}{\left(A^2 - 2A.v.t.\cos\alpha + v^2.t^2\right)^{3/2}} - k.e_1.e_2.v.D.\cos\alpha.\int \frac{t.dt}{\left(A^2 - 2A.v.t.\cos\alpha + v^2.t^2\right)^{3/2}}$$

[I] **[II]**

Suppose that a = A_2 , b = -2A.v.cosα , c = v^2 , X = a + b.t + c.t^2 and q = 4a.c-b^2 , then:

[I] : $\int \frac{dt}{X^{3/2}} = \int \frac{dt}{X.\sqrt{X}} = \frac{2.(2c.t + b)}{q.\sqrt{X}}$; [II] : $\int \frac{t.dt}{X.\sqrt{X}} = -\frac{2.(b.t + 2a)}{q.\sqrt{X}}$,

see Handbook of Chemistry and Physics.

The momentum of f_1 over the period Δt according to **[I]** measures:

$$(P_I): k.e_1.e_2.D.A.\left[\frac{2.(2.v^2.t - 2A.v.\cos\alpha)}{(4.A^2.v^2 - 4.A^2.v^2.\cos^2\alpha) \times (A^2 - 2.A.t.v.\cos\alpha + v^2.t^2)^{1/2}} \right]_{-\Delta t/2}^{\Delta t/2} =$$

$$= -k.e_1.e_2.D.A.\left[\frac{2.(2A.v.\cos\alpha - 2v^2.t)}{4A^2.v^2.\sin^2\alpha.(A^2 - 2A.v.t.\cos\alpha + v^2.t^2)^{1/2}} \right]_{-\Delta t/2}^{\Delta t/2} =$$

$$= k.e_1.e_2.D.A.\left\{ \frac{2.(2A.v.\cos\alpha + v^2.A.\sqrt{2}/c)}{4A^2.v^2.\sin^2\alpha.(A^2 + v.A^2.\sqrt{2}.\cos\alpha/c + v^2.A^2/2c^2)^{1/2}} \right\}$$

$$-k.e_1.e_2.D.A.\left\{ \frac{2.(2A.v.\cos\alpha - v^2.A.\sqrt{2}/c)}{4A^2.v^2.\sin^2\alpha.(A^2 - v.A^2.\sqrt{2}.\cos\alpha/c + v^2.A^2/2c^2)^{1/2}} \right\} =$$

$$= \frac{k.e_1.e_2.D.\left(2\cos\alpha + v.\sqrt{2}/c\right)}{2A.v.\sin^2\alpha.\left(1 + v.\sqrt{2}.\cos\alpha/c + v^2/2c^2\right)^{1/2}} - \frac{k.e_1.e_2.D.\left(2\cos\alpha - v.\sqrt{2}/c\right)}{2A.v.\sin^2\alpha.\left(1 - v.\sqrt{2}\cos\alpha/c + v^2/2c^2\right)^{1/2}},$$

if $\Delta t = \frac{\sqrt{2}.A}{c}$

The momentum of f_1 over the period Δt according to [II] measures:

$$\left(P_{II}\right): k.e_1.e_2.v.D.\cos\alpha.\left[\frac{2.\left(-2A.v.t.\cos\alpha + 2A^2\right)}{\left(4A^2.v^2 - 4A^2.v^2.\cos^2\alpha\right) \times \left(A^2 - 2A.v.t.\cos\alpha + v^2.t^2\right)^{1/2}}\right]_{-\Delta t/2}^{\Delta t/2} =$$

$$= k.e_1.e_2.v.D.\cos\alpha.\left\{\frac{2.\left(2A^2 - A^2.v.\sqrt{2}/c\right)}{4A^2.v^2.\sin^2\alpha.\left(A^2 - A^2.v.\sqrt{2}.\cos\alpha/c + A^2.v^2/2c^2\right)^{1/2}}\right\}$$

$$-k.e_1.e_2.v.D.\cos\alpha.\left\{\frac{2.\left(2A^2 + v.\sqrt{2}.\cos\alpha/c\right)}{4A^2.v^2.\sin^2\alpha.\left(A^2 + A^2.v.\sqrt{2}.\cos\alpha/c + A^2.v^2/2c^2\right)}\right\} =$$

$$= \frac{k.e_1.e_2.D.\cos\alpha.\left(2 - v.\sqrt{2}.\cos\alpha/c\right)}{2A.v.\sin^2\alpha.\left(1 - v.\sqrt{2}.\cos\alpha/c + v^2/2c^2\right)^{1/2}} - \frac{k.e_1.e_2.D.\cos\alpha.\left(2 + v.\sqrt{2}.\cos\alpha/c\right)}{2A.v.\sin^2\alpha.\left(1 + v.\sqrt{2}.\cos\alpha/c + v^2/2c^2\right)^{1/2}}$$

The total momentum of f_1 over Δt is given by $P_I + P_{II}$:

$$P_I + P_{II} = \frac{k.e_1.e_2.v.D.\sqrt{2}/c}{2A.v.\sin^2\alpha.\left(1 + v.\sqrt{2}.\cos\alpha/c + v^2/2c^2\right)^{1/2}} - \frac{k.e_1.e_2.v.D.\sqrt{2}.\cos^2\alpha/c}{2A.v.\sin^2\alpha.\left(1 + \sqrt{2}.\cos\alpha/c + v^2/2c^2\right)^{1/2}}$$

$$+ \frac{k.e_1.e_2.v.D.\sqrt{2}/c}{2A.v.\sin^2\alpha.\left(1 - v.\sqrt{2}.\cos\alpha/c + v^2/2c^2\right)^{1/2}} - \frac{k.e_1.e_2.v.D.\sqrt{2}.\cos^2\alpha/c}{2A.v.\sin^2\alpha.\left(1 - v.\sqrt{2}.\cos\alpha/c + v^2/2c^2\right)^{1/2}}$$

$$= \frac{k.e_1.e_2.v.D.\sqrt{2}.\sin^2\alpha/c}{2A.v.\sin^2\alpha.\left(1 - v.\sqrt{2}.\cos\alpha/c + v^2/2c^2\right)^{1/2}} + \frac{k.e_1.e_2.v.D.\sqrt{2}.\sin^2\alpha/c}{2A.v.\sin^2\alpha.\left(1 + v.\sqrt{2}.\cos\alpha/c + v^2/2c^2\right)^{1/2}}$$

Consequently, the average force along *A* measures:

$$f_{g1} = \frac{1}{\Delta t}\cdot\int_{-\Delta t/2}^{\Delta t/2} f_1.dt = \frac{k.e_1.e_2D}{2A^2.\left(1 - v.\sqrt{2}.\cos\alpha/c + v^2/2c^2\right)^{1/2}} + \frac{k.e_1.e_2D}{2A^2.\left(1 + v.\sqrt{2}.\cos\alpha/c + v^2/2c^2\right)^{1/2}}$$

which can be simplified as is shown below:

$$f_{g1} = \frac{k.e_1.e_2.D}{A^2}\cdot\left\{\frac{1}{2.\left(1 - v.\sqrt{2}.\cos\alpha/c + v^2/2c^2\right)^{1/2}} + \frac{1}{2.\left(1 + v.\sqrt{2}.\cos\alpha/c + v^2/2c^2\right)^{1/2}}\right\}$$

$$= \frac{k.e_1.e_2.D}{A^2}\cdot\left[\frac{(1/2).\left(1 + v.\sqrt{2}.\cos\alpha/c + v^2/2c^2\right)^{1/2} + (1/2).\left(1 - v.\sqrt{2}.\cos\alpha/c + v^2/2c^2\right)^{1/2}}{\left\{\left(1 + v^2/2c^2\right)^2 - \left(v.\sqrt{2}.\cos\alpha/c\right)^2\right\}^2}\right]$$

After the substitution of: $1 + v^2/2c^2 = a$; $v.\sqrt{2}/c = b$; $\sqrt{a} = P$; $b.\sqrt{a}.\cos\alpha/2a = q$ the numerator between [ ] may be expressed by:

$$(1/2).\sqrt{a + b.\cos\alpha} + (1/2).\sqrt{a - b.\cos\alpha} = (1/2).\sqrt{a.(1 + b.\cos\alpha/a)} + (1/2).\sqrt{a.(1 - b.\cos\alpha/a)}$$

$$=(1/2).\sqrt{a.(1+b.\cos\alpha/2a)^2-a.(b.\cos\alpha/2a)^2}+(1/2).\sqrt{a.(1-b.\cos\alpha/2a)^2-a.(b.\cos\alpha/2a)^2}$$

$$=(1/2).\sqrt{(\sqrt{a}+b.\sqrt{a}.\cos\alpha/2a)^2-(b.\sqrt{a}.\cos\alpha/2a)^2}+(1/2).\sqrt{(\sqrt{a}-b.\sqrt{a}.\cos\alpha/2a)^2-(b.\sqrt{a}.\cos\alpha/2a)^2}$$

$$=(1/2).\sqrt{(P+q)^2-q^2}+(1/2).\sqrt{(P-q)^2-q^2}=(1/2).\left\{P+q-\frac{q^2}{2.(P+q)}\right\}+(1/2).\left\{P-q-\frac{q^2}{2.(P-q)}\right\}$$

$$=P-\frac{q^2}{4.(P+q)}-\frac{q^2}{4.(P-q)}=P-\frac{q^2}{4P}\cdot\left(\frac{1}{1+q/P}+\frac{1}{1-q/P}\right)\approx P-\frac{q^2}{2P}$$

$$q^2=\frac{b^2.\cos^2\alpha}{4a}=\frac{2v^2.\cos^2\alpha}{4c^2.(1+v^2/2c^2)}\approx\frac{v^2.\cos^2\alpha}{2c^2}\cdot(1-v^2/2c^2)\ ;\quad P\approx 1+\frac{v^2}{4c^2}$$

$$1+\frac{v^2}{4c^2}-\frac{v^2.\cos^2\alpha}{4c^2}\cdot\left(1-\frac{v^2}{2c^2}\right)\cdot\left(1-\frac{v^2}{4c^2}\right)\approx 1+\frac{v^2}{4c^2}-\frac{v^2}{4c^2}\cdot\cos^2 a$$, thus:

$$f_{g1}=\frac{k.e_1.e_2.D}{A^2}\cdot\left\{\frac{1+v^2/(4c^2)-v^2.\cos^2\alpha/(4c^2)}{\sqrt{1+v^2.(1-2.\cos^2\alpha/c^2)}}\right\}\approx\frac{k.e_1.e_2.D}{A^2}\cdot\left(1+\frac{v^2}{4c^2}-\frac{v^2}{4c^2}.\cos^2\alpha\right)\cdot\left(1-\frac{v^2}{2c^2}+\frac{v^2}{c^2}.\cos^2\alpha\right)$$

$$\approx\frac{k.e_1.e_2.D}{A^2}\cdot\left(1+\frac{v^2}{4c^2}-\frac{v^2}{4c^2}\cdot\cos^2\alpha-\frac{v^2}{2c^2}+\frac{v^2}{c^2}\cos^2\alpha\right)$$

$$f_{g1}\approx\frac{k.e_1.e_2.D}{A^2}\cdot\left(1-\frac{v^2}{4c^2}+\frac{3v^2}{4c^2}\cdot\cos^2\alpha\right)\approx f_0\cdot\left(1+\frac{v^2}{4c^2}+\frac{v^2}{4c^2}\cdot\cos^2\alpha\right)$$

The force perpendicular to *A* can be described by:

$$f_2=\frac{k.e_1.e_2.y.D}{\left\{(A+x)^2+y^2\right\}^{3/2}}=\frac{k.e_1.e_2.D.v.t.\sin\alpha}{(A^2-2A.v.t.\cos\alpha+v^2.t^2)^{3/2}}\ ;$$

$$(x=-v.t.\cos\alpha\ ;\ y=v.t.\sin\alpha\ ;\ \Delta t=A.\sqrt{2}/C)$$

The momentum of f_2 is expressed by:

$$\int f_2.dt=k.e_1.e_2.D.v.\sin\alpha.\int\frac{t.dt}{(A^2-2A.v.t.\cos\alpha+v^2.t^2)^{3/2}}$$

$$\int_{-\Delta t/2}^{\Delta t/2}f_2.dt=-k.e_1.e_2.D.v.\sin\alpha\left[\frac{2.(2A^2-2A.v.t.\cos\alpha)}{(4A^2.v^2-4A^2.v^2.\cos^2\alpha)\times(A^2-2A.v.t.\cos\alpha+v^2.t^2)^{1/2}}\right]_{-\Delta t/2}^{\Delta t/2}$$

$$=k.e_1.e_2.D.v.\sin\alpha\left\{\begin{array}{c}\dfrac{-2.(2A^2-A^2.v.\sqrt{2}.\cos\alpha/c)}{4A^2.v^2.\sin^2\alpha.(A^2-A^2.v.\sqrt{2}.\cos\alpha/c+A^2.v^2/2c^2)^{1/2}}+\\ \dfrac{2.(2A^2+A^2.v.\sqrt{2}.\cos\alpha/c)}{4A^2.v^2.\sin^2\alpha.(A^2+A^2.v.\sqrt{2}.\cos\alpha/c+A^2.v^2/2c^2)^{1/2}}\end{array}\right\}$$

$$=k.e_2.e_2.D.\left\{\frac{2-v.\sqrt{2}.\cos\alpha/c}{2A.v.\sin\alpha.(1-v.\sqrt{2}.\cos\alpha/c+v^2/2c^2)^{1/2}}+\frac{2+v.\sqrt{2}.\cos\alpha/c}{2A.v.\sin\alpha.(1+v.\sqrt{2}.\cos\alpha/c+v^2/2c^2)^{1/2}}\right\}$$

Consequently, the average force perpendicular to *A* measures:

$$f_{g2}=\frac{1}{\Delta t}\cdot\int_{-\Delta t/2}^{\Delta t/2} f_2.dt=\frac{k.e_1.e_2.D}{A^2}\cdot\left\{\begin{array}{l}\dfrac{\cos\alpha}{2.\sin\alpha.\left(1-v.\sqrt{2}.\cos\alpha/c+v^2/2c^2\right)^{1/2}}+\\ \dfrac{\cos\alpha}{2.\sin\alpha.\left(1+v.\sqrt{2}.\cos\alpha/c+v^2/2c^2\right)^{1/2}}\end{array}\right\}$$

$$-\frac{k.e_1.e_2.D}{A^2}\cdot\left\{\begin{array}{l}\dfrac{c}{v.\sqrt{2}.\sin\alpha.\left(1-v.\sqrt{2}.\cos\alpha/c+v^2/2c^2\right)^{1/2}}-\\ \dfrac{c}{v.\sqrt{2}.\sin\alpha.\left(1+v.\sqrt{2}.\cos\alpha/c+v^2/2c^2\right)^{1/2}}\end{array}\right\}$$

If this formula is expressed as:

$$f_{g2}=F.\left\{\frac{\dfrac{\cos\alpha}{\sin\alpha}\cdot\dfrac{1}{2}\cdot\sqrt{1+v.\sqrt{2}.\cos\alpha/c+v^2/2c^2}+\dfrac{\cos\alpha}{\sin\alpha}\cdot\dfrac{1}{2}\sqrt{1-v.\sqrt{2}.\cos\alpha/c+v^2/2c^2}}{\sqrt{\left(1+v^2/2c^2\right)^2-\left(v.\sqrt{2}.\cos\alpha/c\right)^2}}\right\}\quad\textbf{[= I]}$$

$$-F.\left\{\frac{\dfrac{c.\sqrt{2}}{v.\sin\alpha}\cdot\dfrac{1}{2}\cdot\sqrt{1+v.\sqrt{2}.\cos\alpha/c+v^2/2c^2}-\dfrac{c.\sqrt{2}}{v.\sin\alpha}\cdot\dfrac{1}{2}\cdot\sqrt{1-v.\sqrt{2}.\cos\alpha/c+v^2/2c^2}}{\sqrt{\left(1+v^2/2c^2\right)^2-\left(v.\sqrt{2}.\cos\alpha/c\right)^2}}\right\}\quad\textbf{[= II]}$$

in which $F = k.e_1.e_2.D/A^2$,
the numerator of formula [I] measures:

$$I_{numerator}=\frac{1}{2}\cdot\sqrt{\frac{\cos^2\alpha}{\sin^2\alpha}\cdot\left(1+\frac{v^2}{2c^2}\right)+\frac{\cos^3\alpha}{\sin^2\alpha}\cdot\frac{v.\sqrt{2}}{c}}+\frac{1}{2}\cdot\sqrt{\frac{\cos^2\alpha}{\sin^2\alpha}\cdot\left(1+\frac{v^2}{2c^2}\right)-\frac{\cos^3\alpha}{\sin^2\alpha}\cdot\frac{v.\sqrt{2}}{c}}$$

If $\dfrac{\cos^2\alpha}{\sin^2\alpha}\cdot\left(1+\dfrac{v^2}{2c^2}\right)=a$, $\dfrac{\cos^3\alpha}{\sin^2\alpha}\cdot\dfrac{v.\sqrt{2}}{c}=b$, $\sqrt{a}=P$ and $\dfrac{b.\sqrt{a}}{2a}=q$, then:

$$I_{numerator}=(1/2).\sqrt{a+b}+(1/2).\sqrt{a-b}=(1/2).\sqrt{\left(\sqrt{a}+b.\sqrt{a}/2a\right)^2-\left(b.\sqrt{a}/2a\right)^2}$$

$$+(1/2).\sqrt{\left(\sqrt{a}-b.\sqrt{a}/2a\right)^2-\left(b.\sqrt{a}/2a\right)^2}$$

$$=(1/2)\cdot\sqrt{(P+q)^2-q^2}+(1/2)\cdot\sqrt{(P-q)^2-q^2}\approx\frac{1}{2}\cdot\left\{P+q-\frac{q^2}{2.(P+q)}\right\}+\frac{1}{2}\cdot\left\{P-q-\frac{q^2}{2.(P-q)}\right\}$$

$$\approx P-\frac{q^2}{2P}\approx\frac{\cos\alpha}{\sin\alpha}\cdot\left(1+\frac{v^2}{4c^2}\right)-\frac{2v^2.\cos^6\alpha/\left(c^2.\sin^4\alpha\right)}{\left(4.\cos^2\alpha/\sin^2\alpha\right)\cdot\left(1+v^2/2c^2\right)\cdot\left(2.\cos\alpha/\sin\alpha\right)\cdot\left(1+v^2/4c^2\right)}$$

$$\approx \frac{\cos\alpha}{\sin\alpha}\cdot\left(1+\frac{v^2}{4c^2}\right)-\frac{v^2.\cos^3\alpha}{4c^2.\sin\alpha\cdot\left(1+v^2/2c^2\right)\cdot\left(1+v^2/4c^2\right)}$$

$$\approx \frac{\cos\alpha}{\sin\alpha}\cdot\left(1+\frac{v^2}{4c^2}\right)-\frac{v^2.\cos^3\alpha}{4c^2.\sin\alpha}\cdot\left(\frac{1}{1+3v^2/4c^2}\right)$$

$$I_{numerator}\approx \frac{\cos\alpha}{\sin\alpha}\cdot\left(1+\frac{v^2}{4c^2}\right)-\frac{v^2.\cos^3\alpha}{4c^2.\sin\alpha}\left(1-\frac{3v^2}{4c^2}\right)\approx \frac{\cos\alpha}{\sin\alpha}\cdot\left(1+\frac{v^2}{4c^2}\right)-\frac{v^2}{4c^2}\cdot\frac{\cos^3\alpha}{\sin\alpha}$$

$$\approx \frac{\cos\alpha}{\sin\alpha}+\frac{v^2}{4c^2}\cdot\frac{\cos\alpha}{\sin\alpha}-\frac{v^2}{4c^2}\,\frac{\cos^3\alpha}{\sin\alpha}\approx \frac{\cos\alpha}{\sin\alpha}+\frac{v^2}{4c^2}\cdot\frac{\cos\alpha}{\sin\alpha}\cdot\left(1-\cos^2\alpha\right)$$

$$\approx \frac{\cos\alpha}{\sin\alpha}+\frac{v^2}{4c^2}\cdot\sin\alpha.\cos\alpha$$

The numerator of formula [II] measures:

$$II_{numerator}=\frac{-c.\sqrt{2}}{2v.\sin\alpha}\cdot\sqrt{1+\frac{v.\sqrt{2}.\cos\alpha}{c}+\frac{v^2}{2c^2}}+\frac{c.\sqrt{2}}{2v.\sin\alpha}\cdot\sqrt{1-\frac{v.\sqrt{2}.\cos\alpha}{c}+\frac{v^2}{2c^2}}$$

$$=-\frac{1}{2}\cdot\sqrt{\frac{2c^2}{v^2.\sin^2\alpha}\cdot\left(1+\frac{v^2}{2c^2}\right)+\frac{2c.\sqrt{2}.\cos\alpha}{v.\sin^2\alpha}}+\frac{1}{2}\cdot\sqrt{\frac{2c^2}{v^2.\sin^2\alpha}\cdot\left(1+\frac{v^2}{2c^2}\right)-\frac{2c.\sqrt{2}.\cos\alpha}{v.\sin^2\alpha}}$$

After the substitution of:

$\frac{2c^2}{v^2.\sin^2\alpha}\cdot\left(1+\frac{v^2}{2c^2}\right)=a$, $\frac{2c.\sqrt{2}.\cos\alpha}{v.\sin^2\alpha}=b$, $\sqrt{a}=P$ and $\frac{b.\sqrt{a}}{2a}=q$,

the numerator of [II] may be expressed as follows:

$$II_{numerator}=-\frac{1}{2}\cdot\sqrt{a+b}+\frac{1}{2}\cdot\sqrt{a-b}$$

$$=-\frac{1}{2}\cdot\sqrt{\left(\sqrt{a}+\frac{b.\sqrt{a}}{2a}\right)^2-\left(\frac{b.\sqrt{a}}{2a}\right)^2}+\frac{1}{2}\cdot\sqrt{\left(\sqrt{a}-\frac{b.\sqrt{a}}{2a}\right)^2-\left(\frac{b.\sqrt{a}}{2a}\right)^2}$$

$$=-\frac{1}{2}\cdot\sqrt{\left(P+q\right)^2-q^2}+\frac{1}{2}\cdot\sqrt{\left(P-q\right)^2-q^2}$$

$$\approx -\frac{1}{2}\cdot\left\{P+q-\frac{q^2}{2\left(P+q\right)}\right\}+\frac{1}{2}\cdot\left\{P-q-\frac{q^2}{2\left(P-q\right)}\right\}\approx -q-\frac{q^3}{2P^2}$$

Resubstitution gives:

$$q=\frac{c.\sqrt{2}}{v.\sin\alpha}\cdot\left(1+\frac{v^2}{4c^2}\right)\cdot\frac{2c.\sqrt{2}.\cos\alpha}{v.\sin^2\alpha}\cdot\frac{v^2.\sin^2\alpha}{4c^2.\left(1+v^2/2c^2\right)}$$

$$\approx \frac{4c^2.\cos\alpha}{v^2.\sin^3\alpha}\cdot\left(1+v^2/4c^2\right)\cdot\frac{v^2.\sin^2\alpha}{4c^2}\cdot\left(1-v^2/2c^2\right)\approx \frac{\cos\alpha}{\sin\alpha}\cdot\left(1-v^2/4c^2\right)$$

$$\frac{q^3}{2P^2}=\frac{\cos^3\alpha}{\sin^3\alpha}\cdot\left(1-\frac{3v^2}{4c^2}\right)\cdot\frac{v^2.\sin^2\alpha}{4c^2.\left(1+v^2/2c^2\right)}\approx \frac{\cos^3\alpha}{\sin^3\alpha}\cdot\frac{v^2.\sin^2\alpha}{4c^2}\cdot\left(1-\frac{v^2}{2c^2}\right)\approx \frac{v^2}{4c^2}\cdot\frac{\cos^3\alpha}{\sin\alpha}\;;$$

$$-\left(q+\frac{q^3}{2P^2}\right)\approx -\left(\frac{\cos\alpha}{\sin\alpha}-\frac{v^2}{4c^2}\cdot\frac{\cos\alpha}{\sin\alpha}+\frac{v^2}{4c^2}\cdot\frac{\cos^3\alpha}{\sin\alpha}\right)$$

$$\left(I+II\right)_{numerator} \approx \frac{-\cos\alpha}{\sin\alpha}+\frac{v^2}{4c^2}\cdot\frac{\cos\alpha}{\sin\alpha}-\frac{v^2}{4c^2}\cdot\frac{\cos^3\alpha}{\sin\alpha}+\frac{\cos\alpha}{\sin\alpha}+\frac{v^2}{4c^2}\cdot\sin\alpha\cdot\cos\alpha$$

$$\approx \frac{v^2}{4c^2}\cdot\frac{\cos\alpha}{\sin\alpha}\cdot\left(1-\cos^2\alpha\right)+\frac{v^2}{4c^2}\cdot\sin\alpha\cdot\cos\alpha$$

The force perpendicular to *A* can now be described by:

$$f_{g2} \approx \frac{k.e_1.e_2.D}{A^2}\cdot\frac{v^2.\sin\alpha.\cos\alpha/2c^2}{\sqrt{\left(1+v^2/2c^2\right)^2-\left(v.\sqrt{2}.\cos\alpha/c\right)^2}} \approx \frac{k.e_1.e_2.D}{A^2}\cdot\frac{v^2.\sin\alpha.\cos\alpha/2c^2}{\sqrt{1+v^2.\left(1-2.\cos^2\alpha\right)/c^2}}$$

$$\approx \frac{k.e_1.e_2.D}{A^2}\cdot\frac{v^2}{2c^2}\cdot\sin\alpha\cdot\cos\alpha\cdot\left\{1-\frac{v^2}{2c^2}\cdot\left(1-2.\cos^2\alpha\right)\right\}$$,thus

$$f_{g2} \approx \frac{k.e_1.e_2.D}{A^2}\cdot\frac{v^2}{2c^2}\cdot\sin\alpha\cdot\cos\alpha$$

Chapter 3-*pages 19/20 (A_v-axial)*

From *figure 14* can be derived:

$t_1.\left(c-V+v\right)=\left(t_1+t_2\right)\cdot v$; $t_1.\left(c-V\right)+t_1.v=t_1.v+t_2.v$; $t_1=t_2\cdot\frac{v}{\left(c-V\right)}$; $t_2=t_1\cdot\frac{\left(c-V\right)}{v}$;

$\left(t_1+t_2\right)\cdot\left(c-V+v\right)=\left(t_1+t_2-\Delta t_0\right)\cdot\left(c+V\right)+\Delta t_0.v$; $\left(t_1+t_2\right)\cdot\left(c-V+v\right)=\left(t_1+t_2\right)\cdot\left(c+V\right)-\Delta t_0\cdot\left(c+V-v\right)$;

$\left(c-V+v\right)=\left(c+V\right)-\Delta t_0\cdot\frac{\left(c+V-v\right)}{\left(t_1+t_2\right)}$; $\Delta t_0\cdot\frac{\left(c+V-v\right)}{t_1+t_2}=2V-v$; $t_1+t_2=\Delta t_0\cdot\frac{\left(c+V-v\right)}{2V-v}$;

$t_1+t_2\cdot\frac{\left(c-V\right)}{v}=\Delta t_0\cdot\frac{\left(c+V-v\right)}{2V-v}$; $t_1\cdot\frac{\left(c-V+v\right)}{v}=\Delta t_0\cdot\frac{\left(c+V-v\right)}{2V-v}$; $t_2=\Delta t_0\cdot\frac{\left(c+V-v\right)}{\left(2V-v\right)}\cdot\frac{\left(c-V\right)}{\left(c-V+v\right)}$

From the figure can also be derived:

$A_v=t_2.\left(c-V+v\right)-v.\Delta t_v/2$ and $A_v-v.\Delta t_v/2=\left(c+V\right)\cdot\left(t_2-\Delta t_v\right)$, thus:

$A_v=t_2.\left(c+V\right)-\Delta t_v.\left(c+V\right)+\Delta t_v.v/2$, which gives:

$t_2.\left(c-V+v\right)-v.\Delta t_v/2=t_2.\left(c+V\right)-\Delta t_v.\left(c+V\right)+\Delta t_v.v/2$;

$\Delta t_v.v-\Delta t_v.\left(c+V\right)=t_2.\left(c-V+v\right)-t_2.\left(c+V\right)$; $\Delta t_v.\left(c+V-v\right)=t_2.\left(c+V-c+V-v\right)$;

$\Delta t_v=t_2.\frac{\left(2V-v\right)}{\left(c+V-v\right)}$. Uniting the formula of t_2 and the formula of Δ_{tv} gives:

$$\Delta t_v=\Delta t_0\cdot\frac{\left(c+V-v\right)}{\left(2V-v\right)}\cdot\frac{\left(c-V\right)}{\left(c-V+v\right)}\cdot\frac{\left(2V-v\right)}{\left(c+V-v\right)};\ \Delta t_v=\Delta t_0\cdot\frac{\left(c-V\right)}{\left(c-V+v\right)}.$$

If c = 2V = 1, then $\Delta t_v=\Delta t_0\cdot\frac{1}{\left(1+2v\right)}$. Integration of t_1 and Δt_v with A_v makes:

$$A_v=t_2.\left(c-V+v\right)-v.\Delta t_v/2=\Delta t_0\cdot\frac{\left(c+V-v\right)}{\left(2V-v\right)}\cdot\frac{\left(c-V\right)}{\left(c-V+v\right)}\cdot\left(c-V+v\right)-v.\Delta t_0\frac{\left(c-V\right)}{2\left(c-V+v\right)}$$

$$A_v = \Delta t_0 \cdot \frac{(c+V-v)}{(2V-v)} \cdot (c-V) - \frac{v.(c-V)}{2.(c-V+v)}; \quad A_v = \Delta t_0.(c-V).\left\{\frac{(c+V-v)}{(2V-v)} - \frac{v}{2.(c-V+v)}\right\}$$

where $A_0 = \Delta t_0 \cdot \frac{(c+V).(c-V)}{2V}$, thus $\Delta t_0.(c-V) = A_0 \cdot \frac{2V}{(c+V)}$ and:

$$A_v = A_0 \cdot \frac{2V}{(c+V)} \cdot \left\{\frac{(c+V-v)}{(2V-v)} - \frac{v}{2.(c-V+v)}\right\} = A_0 \cdot \frac{V}{(c+V)} \cdot \left\{\frac{2.(c+V).(c-V)+v.(2V-v)}{v.(3V-c)+2V.(c-V)-v^2}\right\}$$

With the substitution of c = 2V and c = 1, the formula of the natural distance becomes:

$$A_v = A_0 \cdot \left(\frac{3+2|v|-2|v|^2}{3+3|v|-6|v|^2}\right)$$

The absolute value of the velocity *v* has to be used in this equation.
An equal result can be obtained from *figure 15*.

$$t_1.(c-V) = t_2.v\,;\; t_1 = t_2 \cdot \frac{v}{(c-V)}\,;\; t_2 = t_1 \cdot \frac{(c-V)}{v}; \quad (t_1+t_2).(c-V) = (t_1+t_2-\Delta t_0).(c+V-v) + \Delta t_0.v$$

$$(t_1+t_2).(c-V) = (t_1+t_2).(c+V-v) - \Delta t_0.(c+V-v+v)\,;\; (c-V) = (c+V-v) - \Delta t_0 \cdot \frac{(c+V)}{(t_1+t_2)}$$

$$\Delta t_0 \cdot \frac{(c+V)}{(t_1+t_2)} = 2V - v\,; \quad t_1 + t_2 = \Delta t_0 \cdot \frac{(c+V)}{(2V-v)}; \quad t_1 + t_1.\frac{(c-V)}{v} = \Delta t_0 \cdot \frac{(c+V)}{(2V-v)};$$

$t_1 \cdot \frac{(c-V+v)}{v} = \Delta t_0 \cdot \frac{(c+V)}{(2V-v)}$; $t_2 = \Delta t_0 \cdot \frac{(c+V)}{(2V-v)} \cdot \frac{(c-V)}{(c-V+v)}$. From the figure can also be derived:

$A_v = t_2.(c-V) + v.\Delta t_v / 2$ and $A_v + v.\Delta t_v / 2 = (c+V-v).(t_2 - \Delta t_v)$, thus:

$A_v = t_2.(c+V-v) - \Delta t_v.(c+V-v) - v.\Delta t_v / 2$, which gives:

$t_2.(c-V) + v.\Delta t_v / 2 = t_2.(c+V-v) - \Delta t_v.(c+V-v) - v.\Delta t_v / 2$;

$\Delta t_v.v + \Delta t_v.(c+V-v) = t_2.(c+V-v) - t_2.(c-V)$; $\Delta t_v.(c+V) = t_2.(2V-v)$; $\Delta t_v = t_2 \cdot \frac{(2V-v)}{(c+V)}$. Uniting the formulas of t_2 and Δt_v gives:

$\Delta t_v = \Delta t_0 \cdot \frac{(c+V)}{(2V-v)} \cdot \frac{(c-V)}{(c-V+v)} \cdot \frac{(2V-v)}{(c+V)}$; $\Delta t_v = \Delta t_0 \cdot \frac{(c-V)}{(c-V+v)}$ or

$\Delta t_v = \Delta t_0 \cdot \frac{1}{(1+2v)}$, if c = 2V = 1.

Integration of $t_2 + \Delta t_v$ with A_v makes:

$$A_v = t_2.(c-V) + v.\Delta t_v / 2 = \Delta t_0 \cdot \frac{(c+V)}{(2V-v)} \cdot \frac{(c-V)}{(c-V+v)} \cdot (c-V) + \Delta t_0 \cdot \frac{v.(c-V)}{2(c-V+v)}$$

$A_v = \Delta t_0 \cdot \frac{(c-V)}{(c-V+v)} \cdot \left\{\frac{(c+V)}{(2V-v)} \cdot (c-V) + \frac{v}{2}\right\}$; $A_0 = \Delta t_0 \cdot \frac{(c+V).(c-V)}{2V}$, or

$\Delta t_0 \cdot (c-V) = A_0 \cdot \frac{2V}{(c+V)}$, thus: $A_v = A_0 \cdot \frac{2V}{(c+V)} \cdot \left\{\frac{(c+V).(c-V)}{(2V-v).(c-V+v)} + \frac{v}{2.(c-V+v)}\right\}$

$$= A_0 \cdot \frac{V}{(c+V)} \cdot \left\{ \frac{2.(c+V).(c-V) + v.(2V-v)}{v.(3V-c) + 2V.(c-V) - v^2} \right\}$$

which proves the equality of the results obtained from both figures.

Chapter 3-*pages 21/23 (A_v-transversal)*

From *figure 17* can be derived:

$t_1 / v = (t_1 + t_2).(c-V)$; $t_1 = t_1.v.(c-V) + t_2.v.(c-V)$; $t_1.\{1 - v.(c-V)\} = t_2.v.(c-V)$;

$t_1 = t_2 \cdot \frac{v.(c-V)}{1 - v.(c-V)}$; $A_v = t_2.(c-V)$; $\Delta t_v = \Delta t_0 \cdot \frac{t_2}{(t_1 + t_2)}$; $c_v = \frac{\sqrt{a^2 - 1}}{a} = \sqrt{1 - \frac{1}{a^2}} = \sqrt{1 - v^2}$;

$(t_1 + t_2 - A_0).c_v = \frac{t_1}{v} = (t_1 + t_2 - 1).\sqrt{1 - v^2}$; $t_1.\sqrt{1 - v^2} - t_1 / v = (1 - t_2).\sqrt{1 - v^2}$;

$t_1 \cdot \frac{v.\sqrt{1 - v^2} - 1}{v} = \sqrt{1 - v^2} - t_2.\sqrt{1 - v^2}$; $t_1 = \frac{v.\sqrt{1 - v^2}}{v.\sqrt{1 - v^2} - 1} - t_2 \cdot \frac{v.\sqrt{1 - v^2}}{v.\sqrt{1 - v^2} - 1}$;

$t_2 \cdot \frac{v.(c-V)}{1 - v.(c-V)} = \frac{v.\sqrt{1 - v^2}}{v.\sqrt{1 - v^2} - 1} - t_2 \cdot \frac{v.\sqrt{1 - v^2}}{v.\sqrt{1 - v^2} - 1}$; $t_2 \cdot \left\{ \frac{v.(c-V)}{1 - v.(c-V)} + \frac{v.\sqrt{1 - v^2}}{v.\sqrt{1 - v^2} - 1} \right\} = \frac{v.\sqrt{1 - v^2}}{v.\sqrt{1 - v^2} - 1}$;

$$t_2 \cdot \left\{ \frac{v^2.(c-V).\sqrt{1 - v^2} - v.(c-V) + v.\sqrt{1 - v^2} - v^2.(c-V).\sqrt{1 - v^2}}{v.\sqrt{1 - v^2} - 1 - v^2.(c-V).\sqrt{1 - v^2} + v.(c-V)} \right\} = \frac{v.\sqrt{1 - v^2}}{v.\sqrt{1 - v^2} - 1}$$

$t_2 = \frac{v.\sqrt{1 - v^2}.\{1 - v.(c-V)\} - \{1 - v.(c-V)\}}{\sqrt{1 - v^2} - (c-V)} \cdot \frac{\sqrt{1 - v^2}}{v.\sqrt{1 - v^2} - 1}$; $t_2 = \frac{\{1 - v.(c-V)\}.\sqrt{1 - v^2}}{\sqrt{1 - v^2} - (c-V)}$

$t_1 = \frac{\{1 - v.(c-V)\}.\sqrt{1 - v^2}}{\sqrt{1 - v^2} - (c-V)} \cdot \frac{v.(c-V)}{1 - v.(c-V)}$; $t_1 = \frac{v.(c-V).\sqrt{1 - v^2}}{\sqrt{1 - v^2} - (c-V)}$

$A_v = \frac{\{1 - v.(c-V)\}.\sqrt{1 - v^2}.(c-V)}{\sqrt{1 - v^2} - (c-V)}$; $A_v = \frac{(1 - v/2).\sqrt{1 - v^2}}{2.\sqrt{1 - v^2} - 1}$, where c = 1 and V = 1/2

$\Delta t_v = \Delta t_0 \cdot \frac{t_2}{(t_1 + t_2)}$; $t_1 + t_2 = \frac{v.(c-V).\sqrt{1 - v^2}}{\sqrt{1 - v^2} - (c-V)} + \frac{\{1 - v.(c-V)\}.\sqrt{1 - v^2}}{\sqrt{1 - v^2} - (c-V)} = \frac{\sqrt{1 - v^2}}{\sqrt{1 - v^2} - (c-V)}$

$$\frac{t_1}{(t_1 + t_2)} = \frac{\{1 - v.(c-V)\}.\sqrt{1 - v^2}}{\sqrt{1 - v^2} - (c-V)} \cdot \frac{\sqrt{1 - v^2} - (c-V)}{\sqrt{1 - v^2}} = 1 - v.(c-V)$$

$\Delta t_v = \frac{2A_0.V}{(c+V).(c-V)} \cdot \{1 - v.(c-V)\}$; $\Delta t_v \approx \Delta t_0 \cdot \left(1 - \frac{v}{2}\right)$, where c = 1 and V = 1/2.

Read v as v_d.

The absolute value of the velocity v has also to be used in these equations.

Chapter 4-*solution of some integrals.*

Say: n/N = tanφ ; dn/d.tanφ = N; $\tan^2\varphi$ +1 = 1/$\cos^2\varphi$; d.tanφ = dφ /$\cos^2\varphi$; sinφ = n$(n^2 + N^2)^{-1/2}$, cosφ = N$(n^2 + N^2)^{-1/2}$, then:

I

$$y = \int (n^2 + N^2)^{-5/2}.dn$$

$$y = \int \left\{ N^2.\left(\frac{n^2}{N^2} + 1\right)\right\}^{-5/2}.dn$$

$$y = N^{-4}.\int (\tan^2\varphi + 1)^{-5/2} d.\tan\varphi$$

$$y = N^{-4}.\int \left(\frac{1}{\cos^2\varphi}\right)^{-5/2}.\frac{1}{\cos^2\varphi}.d\varphi$$

$$y = N^{-4}.\int \cos^3\varphi.d\varphi$$

$$y = (1/3).N^{-4}.\sin\varphi.(\cos^2\varphi + 2)*$$

$$y = (1/3).N^{-4}.\sin\varphi.(1 - \sin^2\varphi + 2)$$

$$y = N^{-4}.\sin\varphi - N^{-4}.\sin^3\varphi / 3$$

$$y = N^{-4}.n.(n^2 + N^2)^{-1/2} - N^{-4}.n^3.(n^2 + N^2)^{-3/2} / 3$$

II

$$y = \int n.(n^2 + N^2)^{-5/2}.dn$$

$$y = \int n.\left\{ N^2.\left(\frac{n^2}{N^2} + 1\right)\right\}^{-5/2}.dn$$

$$y = N^{-4}.\int n.(\tan^2\varphi + 1)^{-5/2} d.\tan\varphi$$

$$y = N^{-3}.\int \tan\varphi.(\tan^2\varphi + 1)^{-5/2}.d.\tan\varphi$$

$$y = N^{-3}.\int \cos^5\varphi.\frac{\sin\varphi}{\cos\varphi}\cdot\frac{1}{\cos^2\varphi}\cdot d\varphi$$

$$y = N^{-3}.\int \cos^2\varphi.\sin\varphi.d\varphi$$

$$y = N^{-3}.(-\cos^3\varphi / 3)*$$

$$y = N^{-3}.\left\{-N^3.(n^2 + N^2)^{-3/2}\right\} / 3$$

$$y = -(n^2 + N^2)^{-3/2} / 3$$

III

$$y = \int n^2.(n^2 + N^2)^{-5/2}.dn$$

$$y = \int n^2.\left\{ N^2.\left(\frac{n^2}{N^2} + 1\right)\right\}^{-5/2}.dn$$

$$y = N^{-4}.\int n^2.(\tan^2\varphi + 1)^{-5/2} d.\tan\varphi$$

$$y = N^{-2}.\int \tan^2\varphi.(\tan^2\varphi + 1)^{-5/2} d.\tan\varphi$$

$$y = N^{-2}.\int \frac{\sin^2\varphi}{\cos^2\varphi}\cdot\frac{\cos^5\varphi}{\cos^2\varphi}\cdot d\varphi$$

$$y = N^{-2}.\int \sin^2\varphi.\cos\varphi.d\varphi$$

$$y = N^{-2}.\sin^3\varphi / 3*$$

$$y = N^{-2}.n^3.(n^2 + N^2)^{-3/2} / 3$$

IV

$$y = \int n^3.(n^2 + n^2)^{-5/2}.dn$$

$$y = n^3.\left\{ N^2.\left(\frac{n^2}{N^2} + 1\right)\right\}^{-5/2}.dn$$

$$y = N^{-4}.\int n^3.(\tan^2\varphi + 1)^{-5/2}.d.\tan\varphi$$

$$y = N^{-1}.\int \tan^3\varphi.(\tan^2\varphi + 1)^{-5/2} d.\tan\varphi$$

$$y = N^{-1}.\int \frac{\sin^3\varphi}{\cos^3\varphi}\cdot\frac{\cos^5\varphi}{\cos^2\varphi}\cdot d\varphi$$

$$y = N^{-1}.\int \sin^3\varphi.d\varphi$$

$$y = -N^{-1}.\cos\varphi.(\sin^2\varphi + 2) / 3*$$

$$y = -(n^2 + N^2)^{-1/2}.\left\{n^2.(n^2 + N^2)^{-1} + 2\right\} / 3$$

$$y = -n^2.(n^2 + N^2)^{-3/2} / 3 - 2.(n^2 + N^2)^{-1/2} / 3$$

∗ : according to the table in the Handbook of Chemistry and Physics.

Chapter 4- *pages 36/39 (f_a)*

$$\int_{-n}^{n} f_a.dn = \frac{e_1.e_2.k.N}{d^2}\cdot\int_{-n}^{n}\frac{1}{\left(n^2+N^2\right)^{3/2}}\cdot\left\{1+\frac{v^2}{4c^2}+\frac{3v^2}{4c^2}\cdot\frac{n^2}{\left(n^2+N^2\right)}\right\}\cdot dn$$

(I): $$\frac{e_1.e_2.k.N}{d^2}\cdot\int_{-n}^{n}\frac{dn}{\left(n^2+N^2\right)^{3/2}} = \frac{2.e_1.e_2.k.}{d^2.N}\cdot\frac{n}{\left(n^2+N^2\right)^{1/2}};k=\left(4\pi\varepsilon_0\right)^{-1}$$

(II): $$\frac{e_1.e_2.k.N}{d^2}\cdot\frac{v^2}{4c^2}\cdot\int_{-n}^{n}\frac{dn}{\left(n^2+N^2\right)^{3/2}} = \frac{e_1.e_2.k.v^2}{2.d^2.c^2.N}\cdot\frac{n}{\left(n^2+N^2\right)^{1/2}}$$

(III): $$\frac{e_1.e_2.k.N}{d^2}\cdot\frac{3v^2}{4c^2}\cdot\int_{-n}^{n}\frac{n^2.dn}{\left(n^2+N^2\right)^{5/2}} = \frac{e_1.e_2.k.N}{d^2}\cdot\frac{3v^2}{4c^2}\cdot\left[\frac{n^3}{3N^2.\left(n^2+N^2\right)^{3/2}}\right]_{-n}^{n}$$

$$= \frac{e_1.e_2.k.N}{d^2}\cdot\frac{3v^2}{4c^2}\cdot\left\{\frac{2n^3}{3N^2.\left(n^2+N^2\right)^{3/2}}\right\} = \frac{e_1.e_2.k.v^2}{2.d^2.c^2.N}\cdot\frac{n^3}{\left(n^2+N^2\right)^{3/2}}$$

Addition of *I*, *II* and *III* gives:

$$f_{a(1/n)} = \int_{-n}^{n} f_a.dn = \frac{2.e_1.e_2.k.}{d^2}\cdot\frac{n}{N\cdot\left(n^2+N^2\right)^{1/2}} + \frac{e_1.e_2.k.v^2}{2.d^2.c^2.N}\cdot\left\{\left(\frac{n^2}{n^2+N^2}\right)^{1/2}+\left(\frac{n^2}{n^2+N^2}\right)^{3/2}\right\}$$

In the same way can be proved that: $f_{b(1/n)} = 0$.

Chapter 4- *page 44/45 ($f_{br(1/n)}$)*

$\sin(\alpha + \beta) = \sin\alpha .\cos\beta + \cos\alpha .\sin\beta = N.(n^2 + N^2)^{-1/2}$
$\cos(\alpha + \beta) = \cos\alpha .\cos\beta - \sin\alpha .\sin\beta = n.(n^2 + N^2)^{-1/2}$

$$\sin\beta = \frac{V_n}{V_R};\cos\beta = \frac{v}{V_R};\frac{\sin\beta}{\cos\beta}+\frac{\cos\beta}{\sin\beta} = \frac{\sin^2\beta+\cos^2\beta}{\cos\beta.\sin\beta} = \frac{1}{\cos\beta.\sin\beta}; \quad \sin\alpha+\cos\alpha.\frac{\sin\beta}{\cos\beta} = N.\frac{\left(n^2+N^2\right)^{-1/2}}{\cos\beta};$$

$$-\sin\alpha+\cos\alpha\cdot\frac{\cos\beta}{\sin\beta} = n\cdot\frac{\left(n^2+N^2\right)^{-1/2}}{\sin\beta}, \text{ thus } \cos\alpha\cdot\left(\frac{\sin\beta}{\cos\beta}+\frac{\cos\beta}{\sin\beta}\right) = \frac{N.\left(n^2+N^2\right)^{-1/2}}{\cos\beta} = \frac{n.\left(n^2+N^2\right)^{-1/2}}{\sin\beta}$$

$$\cos\alpha = \sin\beta.N.\left(n^2+N^2\right)^{-1/2} + \cos\beta.n.\left(n^2+N^2\right)^{-1/2} = \frac{V_n}{V_R}\cdot N.\left(n^2+N^2\right)^{-1/2} + \frac{v}{V_R}\cdot n.\left(n^2+N^2\right)^{-1/2}$$

In the same way can be found:

$$\sin\alpha = \cos\beta.N.\left(n^2+N^2\right)^{-1/2} - \sin\beta.n.\left(n^2+N^2\right)^{-1/2} = \frac{v}{V_R}\cdot N.\left(n^2+N^2\right)^{-1/2} - \frac{V_n}{V_R}\cdot n.\left(n^2+N^2\right)^{-1/2}$$

The forces along the conductor are expressed by the equation:

$$f_{br(1/n)} = \int_{-n}^{n} f_b.dn = \underbrace{\int_{-n}^{n} f_{g1}.n.\left(n^2+N^2\right)^{-1/2}.dn}_{\textit{I}} + \underbrace{\int_{-n}^{n} f_{g2}.N.\left(n^2+N^2\right)^{-1/2}.dn}_{\textit{II}}$$

where: $$f_{g1} = \frac{e_1.e_2.k}{A^2}\cdot\left(\underset{(1)}{1}\underset{(2)}{-\frac{V_R^{\,2}}{4c^2}}\underset{(3)}{+\frac{V_R^{\,2}}{2c^2}\cdot\sin^2\alpha}\underset{(4)}{+\frac{3V_R^{\,2}}{4c^2}\cdot\cos^2\alpha}\right)$$

$$f_{g2} = -\frac{e_1.e_2.k}{A^2}\cdot\frac{V_R^{\,2}}{2c^2}\cdot\sin\alpha.\cos\alpha \;\;;\;\; A^2 = \left(n^2+N^2\right).d^2 \;\;;\;\; k = \left(4\pi.\varepsilon_0\right)^{-1}$$

I(1): $$\int_{-n}^{n}\frac{e_1.e_2.k}{A^2}\cdot\frac{n.dn}{\left(n^2+N^2\right)^{1/2}} = \frac{e_1.e_2.k}{d^2}\cdot\int_{-n}^{n}\frac{n.dn}{\left(n^2+N^2\right)^{3/2}} = \frac{-e_1.e_2.k}{d^2}\cdot\left[\frac{1}{\left(n^2+N^2\right)^{1/2}}\right]_{-n}^{n} = 0$$

I(2) : $$-\int_{-n}^{n}\frac{e_1.e_2.k}{A^2}\cdot\frac{V_R^{\,2}}{4c^2}\cdot\frac{n.dn}{\left(n^2+N^2\right)^{1/2}} = -\frac{e_1.e_2.k.V_R^{\,2}}{4d^2.c^2}\cdot\int_{-n}^{n}\frac{n.dn}{\left(n^2+N^2\right)^{3/2}} = 0$$

I(3):

$$\int_{-n}^{n}\frac{e_1.e_2.k}{A^2}\cdot\frac{V_R^{\,2}}{2c^2}\cdot\frac{n}{\left(n^2+N^2\right)^{1/2}}\cdot\left\{\frac{v^2}{V_R^{\,2}}\cdot\frac{N^2}{\left(n^2+N^2\right)} + \frac{V_n^{\,2}}{V_R^{\,2}}\cdot\frac{n^2}{\left(n^2+N^2\right)} - \frac{2.v.V_n}{V_R^{\,2}}\cdot\frac{N.n}{\left(n^2+N^2\right)}\right\}\cdot dn$$

a b c

a: $$\int_{-n}^{n}\frac{e_1.e_2.k}{A^2}\cdot\frac{V_R^{\,2}}{2c^2}\cdot\frac{v^2}{V_R^{\,2}}\cdot\frac{N^2.n.dn}{\left(n^2+N^2\right)^{3/2}} = \frac{e_1.e_2.v^2.k.N^2}{2.d^2.c^2}\cdot\int_{-n}^{n}\frac{n.dn}{\left(n^2+N^2\right)^{5/2}} = 0$$

b: $$\int_{-n}^{n}\frac{e_1.e_2.k}{A^2}\cdot\frac{V_R^{\,2}}{2c^2}\cdot\frac{V_n^{\,2}}{V_R^{\,2}}\cdot\frac{n^3.dn}{\left(n^2+N^2\right)^{3/2}} = \frac{e_1.e_2.k.V_n^{\,2}}{2.d^2.c^2}\cdot\int_{-n}^{n}\frac{n^3.dn}{\left(n^2+N^2\right)^{5/2}} = 0$$

c: $$-2.\int_{-n}^{n}\frac{e_1.e_2.k}{A^2}\cdot\frac{V_R^{\,2}}{2c^2}\cdot\frac{v.V_n}{V_R^{\,2}}\cdot\frac{N.n^2.dn}{\left(n^2+N^2\right)^{3/2}} = -\frac{2.e_1.e_2.k.v.V_n.N}{2.d^2.c^2}\cdot\int_{-n}^{n}\frac{n^2.dn}{\left(n^2+N^2\right)^{5/2}} = -\frac{2.e_1.e_2.k.v.V_n}{3.d^2.c^2.N}\cdot\frac{n^3}{\left(n^2+N^2\right)^{3/2}}$$

I(4): $$\int_{-n}^{n}\frac{e_1.e_2.k}{A^2}\cdot\frac{3V_R^{\,2}}{4c^2}\cdot\frac{n}{\left(n^2+N^2\right)^{1/2}}\cdot\left\{\frac{V_n^{\,2}}{V_R^{\,2}}\cdot\frac{N^2}{\left(n^2+N^2\right)} + \frac{v^2}{V_R^{\,2}}\cdot\frac{n^2}{\left(n^2+N^2\right)} + \frac{2v.V_n}{V_R^{\,2}}\cdot\frac{n.N}{\left(n^2+N^2\right)}\right\}\cdot dn$$

a b c

a = 0 en b = 0, see I(3).

c: $$2.\int_{-n}^{n}\frac{e_1.e_2.k}{A^2}\cdot\frac{3V_R^{\,2}}{4c^2}\cdot\frac{v.V_n}{V_R^{\,2}}\cdot\frac{N.n^2.dn}{\left(n^2+N^2\right)^{3/2}} = \frac{3.e_1.e_2.k.v.V_n.N}{2.d^2.c^2}\cdot\int_{-n}^{n}\frac{n^2.dn}{\left(n^2+N^2\right)^{5/2}}$$

$$= \frac{e_1.e_2.k.v.V_n}{2.d^2.c^2.N}\cdot\frac{n^3}{\left(n^2+N^2\right)^{3/2}} \;\;;\;\; I_{total} = \frac{e_1.e_2.k.v.V_n}{6.d^2.c^2.N}\cdot\frac{n^3}{\left(n^2+N^2\right)^{3/2}}$$

II: $$-\int_{-n}^{n}\frac{e_1.e_2.k.V_R^{\,2}N}{2A^2.c^2.\left(n^2+N^2\right)^{1/2}}\cdot\left\{\frac{V_n.v.N^2}{V_R^{\,2}.\left(n^2+N^2\right)} - \frac{V_n^{\,2}.N.n}{V_R^{\,2}.\left(n^2+N^2\right)} + \frac{v^2.N.n}{V_R^{\,2}.\left(n^2+N^2\right)} - \frac{V_n.v.n^2}{V_R^{\,2}.\left(n^2+N^2\right)}\right\}.dn$$

a b c d

a: $$-\int_{-n}^{n}\frac{e_1.e_2.k}{A^2}\cdot\frac{V_R^{\,2}}{2c^2}\cdot\frac{V_n.v}{V_R^{\,2}}\cdot\frac{N^3}{\left(n^2+N^2\right)^{3/2}}\cdot dn = -\frac{e_1.e_2.k.V_n.v.N^3}{2.d^2.c^2}\cdot\int_{-n}^{n}\frac{dn}{\left(n^2+N^2\right)^{5/2}}$$

$$= -\frac{e_1.e_2.k.V_n.v}{d^2.c^2.N} \cdot \frac{n}{\left(n^2+N^2\right)^{1/2}} + \frac{e_1.e_2.k.V_n.v}{3.d^2.c^2.N} \cdot \frac{n^3}{\left(n^2+N^2\right)^{3/2}}$$

b: $$\int_{-n}^{n} \frac{e_1.e_2.k}{A^2} \cdot \frac{V_R^{\,2}}{2c^2} \cdot \frac{V_n^{\,2}}{V_R^{\,2}} \cdot \frac{N^2.n}{\left(n^2+N^2\right)^{3/2}} \cdot dn = \frac{e_1.e_2.k.V_n^{\,2}.N^2}{2.d^2.c^2} \cdot \int_{-n}^{n} \frac{n.dn}{\left(n^2+N^2\right)^{5/2}} = 0$$

c: = 0, see before.

d: $$\int_{-n}^{n} \frac{e_1.e_2.k}{A^2} \cdot \frac{V_R^{\,2}}{2c^2} \cdot \frac{V_n.v}{V_R^{\,2}} \cdot \frac{N.n^2.dn}{\left(n^2+N^2\right)^{3/2}} = \frac{e_1.e_2.k.V_n.v.N}{2.d^2.c^2} \int_{-n}^{n} \frac{n^2.dn}{\left(n^2+N^2\right)^{5/2}} = \frac{e_1.e_2.k.V_n.v}{6.d^2.c^2.N} \cdot \frac{n^3}{\left(n^2+N^2\right)^{3/2}}$$

I + II(total) : $$\int_{-n}^{n} f_b.dn = \frac{2.e_1.e_2.k.V_n.v}{3.d^2.c^2.N} \cdot \frac{n^3}{\left(n^2+N^2\right)^{3/2}} - \frac{e_1.e_2.k.V_n.v}{d^2.c^2.N} \cdot \frac{n}{\left(n^2+N^2\right)^{1/2}}$$

$$f_{br(1/n)} = -\frac{e_1.e_2.k.V_n.v}{d^2.c^2.N} \cdot \left\{ \left(\frac{n^2}{n^2+N^2} \right)^{1/2} - \frac{2}{3} \cdot \left(\frac{n^2}{n^2+N^2} \right)^{3/2} \right\}$$

Chapter 4-*page 45* ($f_{br(n/n)}$)

See page viii.

I(1): $$\frac{e_1.e_2.k}{d^2} \cdot \int_0^n \int_0^n \frac{n.dn}{\left(n^2+N^2\right)^{3/2}} = -\frac{e_1.e_2.k}{d^2} \cdot \int_0^n \frac{n.dn}{\left(n^2+N^2\right)^{1/2}} = -\frac{e_1.e_2.k}{d^2} \cdot \left[\ln\left(n+\sqrt{n^2+N^2}\right) \right]_0^n$$

$$= -\frac{\mathbf{e_1.e_2.k}}{\mathbf{d^2}} \cdot \mathbf{ln}\left(\mathbf{n}+\sqrt{\mathbf{n^2}+\mathbf{N^2}}\right) + \frac{\mathbf{e_1.e_2.k}}{\mathbf{d^2}} \cdot \mathbf{ln}(\mathbf{N})$$

for $$\int_{-n}^{0}\int_{-n}^{0} : -\frac{\mathbf{e_1.e_2.k}}{\mathbf{d^2}} \cdot \mathbf{ln}\left(\mathbf{-n}+\sqrt{\mathbf{n^2}+\mathbf{N^2}}\right) + \frac{\mathbf{e_1.e_2.k}}{\mathbf{d^2}} \cdot \mathbf{ln}(\mathbf{N})$$

I(2): $$-\frac{e_1.e_2.V_R^{\,2}.k}{4.d^2.c^2} \cdot \int_0^n \int_0^n \frac{n.dn}{\left(n^2+N^2\right)^{3/2}} = \frac{\mathbf{e_1.e_2.V_R^{\,2}.k}}{\mathbf{4.d^2.c^2}} \cdot \mathbf{ln}\left(\mathbf{n}+\sqrt{\mathbf{n^2}+\mathbf{N^2}}\right) - \frac{\mathbf{e_1.e_2.V_R^{\,2}.k}}{\mathbf{4.d^2.c^2}} \mathbf{ln}(\mathbf{N})$$

for $$\int_{-\mathbf{n}}^{\mathbf{0}}\int_{-\mathbf{n}}^{\mathbf{0}} : \frac{\mathbf{e_1.e_2.V_R^{\,2}.k}}{\mathbf{4.d^2.c^2}} \cdot \mathbf{ln}\left(-\mathbf{n}+\sqrt{\mathbf{n^2}+\mathbf{N^2}}\right) - \frac{\mathbf{e_1.e_2.V_R^{\,2}.k}}{\mathbf{4.d^2.c^2}} \cdot \mathbf{ln}(\mathbf{N})$$

I(3)a: $$\frac{e_1.e_2.v^2.k.N^2}{2.d^2.c^2} \cdot \int_0^n \int_0^n \frac{n.dn}{\left(n^2+N^2\right)^{5/2}} = -\frac{e_1.e_2.v^2.k.N^2}{6.d^2.c^2} \cdot \int_0^n \frac{dn}{\left(n^2+N^2\right)^{3/2}} = -\frac{e_1.e_2.v^2.k}{6.d^2.c^2} \cdot \left[\frac{n}{\left(n^2+N^2\right)^{1/2}} \right]_0^n = -\frac{e_1.e_2.v^2.k}{6.d^2.c^2} \cdot \frac{n}{\left(n^2+N^2\right)^{1/2}}$$

for $$\int_{-n}^{0}\int_{-n}^{0} : + \frac{e_1.e_2.v^2.k}{6.d^2.c^2} \cdot \frac{n}{\left(n^2+N^2\right)^{1/2}}$$, thus $\sum_{\mathbf{I(3)a}} = \mathbf{0}$.

I(3)b: $$\frac{e_1.e_2.V_n^{\,2}.k}{2.d^2.c^2} \cdot \int_0^n \int_0^n \frac{n^3.dn}{\left(n^2+N^2\right)^{5/2}} = -\frac{e_1.e_2.V_n^{\,2}.k}{6.d^2.c^2} \cdot \left[\int_0^n \frac{n^2.dn}{\left(n^2+N^2\right)^{3/2}} + 2.\int_0^n \frac{dn}{\left(n^2+N^2\right)^{1/2}} \right]$$

$$= -\frac{e_1.e_2.V_n^2.k}{6.d^2.c^2}\cdot\left[\frac{-n}{\left(n^2+N^2\right)^{1/2}}+\ln\left(n+\sqrt{n^2+N^2}\right)\right]_0^n - \frac{e_1.e_2.V_n^2.k}{3.d^2.c^2}\cdot\left[\ln\left(n+\sqrt{n^2+N^2}\right)\right]_0^n$$

$$= -\frac{\mathbf{e_1.e_2.V_n^2.k}}{\mathbf{6.d^2.c^2}}\cdot\left\{\mathbf{\ln\left(n+\sqrt{n^2+N^2}\right)-\ln(N)+2.\ln\left(n+\sqrt{n^2+N^2}\right)-2.\ln(N)}\right\}$$

for $\int\limits_{-n}^{0}\int\limits_{-n}^{0}$: $-\frac{\mathbf{e_1.e_2.V_n^2.k}}{\mathbf{6.d^2.c^2}}\cdot\left\{\mathbf{\ln\left(-n+\sqrt{n^2+N^2}\right)+2.\ln\left(-n+\sqrt{n^2+N^2}\right)-\ln(N)-2.\ln(N)}\right\}$

I(3)c: $-\frac{e_1.e_2.V_n.v.k.N}{d^2.c^2}\int\limits_0^n\int\limits_0^n\frac{n^2.dn}{\left(n^2+N^2\right)^{5/2}} = -\frac{e_1.e_2.V_n.v.k}{3.d^2.c^2.N}\cdot\int\limits_0^n\frac{n^3.dn}{\left(n^2+N^2\right)^{3/2}}$

$$= -\frac{e_1.e_2.V_n.v.k}{3.d^2.c^2.N}\cdot\left[\sqrt{n^2+N^2}\right]_0^n - \frac{e_1.e_2.V_n.v.k.N}{3.d^2.c^2}\cdot\left[\frac{1}{\sqrt{n^2+N^2}}\right]_0^n$$

$$= \frac{\mathbf{2}}{\mathbf{3}}\cdot\frac{\mathbf{e_1.e_2.V_n.v.k}}{\mathbf{d^2.c^2}}\cdot\left\{-\frac{\left(\sqrt{\mathbf{n^2+N^2}}\right)}{\mathbf{N}}+\mathbf{1}-\frac{\mathbf{N}}{\sqrt{\mathbf{n^2+N^2}}}+\mathbf{1}\right\}$$

for $\int\limits_0^n\int\limits_0^n$ **as well as for** $\int\limits_{-n}^{0}\int\limits_{-n}^{0}$

I(4)a: $\frac{3.e_1.e_2.V_n^2.k.N^2}{4.d^2.c^2}\cdot\int\limits_0^n\int\limits_0^n\frac{n.dn}{\left(n^2+N^2\right)^{5/2}} = -\frac{e_1.e_2.V_n^2.k.N^2}{4.d^2.c^2}\cdot\int\limits_0^n\frac{dn}{\left(n^2+N^2\right)^{3/2}} = -\frac{e_1.e_2.V_n^2.k}{4.d^2.c^2}\cdot\left[\frac{n}{\left(n^2+N^2\right)^{1/2}}\right]_0^{n(-n)}$

$$= \pm\frac{e_1.e_2.V_n^2.k}{4.d^2.c^2}\cdot\frac{n}{\left(n^2+N^2\right)^{1/2}}\quad;\quad \sum_{\mathbf{I(4)a}}\mathbf{=0}$$

I(4)b: $\frac{3.e_1.e_2.v^2.k}{4.d^2.c^2}\cdot\int\limits_0^n\int\limits_0^n\frac{n^3.dn}{\left(n^2+N^2\right)^{5/2}} = -\frac{e_1.e_2.v^2.k}{4.d^2.c^2}\cdot\left\{\int\limits_0^n\frac{n^2.dn}{\left(n^2+N^2\right)^{3/2}}+2.\int\limits_0^n\frac{dn}{\left(n^2+N^2\right)^{1/2}}\right\}$

$$= -\frac{e_1.e_2.v^2.k}{4.d^2.c^2}\cdot\left[\frac{-n}{\left(n^2+N^2\right)^{1/2}}+\ln\left(n+\sqrt{n^2+N^2}\right)\right]_0^n - \frac{e_1.e_2.v^2.k}{2.d^2.c^2}\cdot\left[\ln\left(n+\sqrt{n^2+N^2}\right)\right]_0^n$$

$$= -\frac{\mathbf{e_1.e_2.v^2.k}}{\mathbf{4.d^2.c^2}}\cdot\left\{\mathbf{\ln\left(n+\sqrt{n^2+N^2}\right)-\ln(N)+2.\ln\left(n+\sqrt{n^2+N^2}\right)-2.\ln(N)}\right\}$$

for $\int\limits_{-n}^{0}\int\limits_{-n}^{0}$: $-\frac{\mathbf{e_1.e_2.v^2.k}}{\mathbf{4.d^2.c^2}}\cdot\left\{\mathbf{\ln\left(-n+\sqrt{n^2+N^2}\right)-\ln(N)+2.\ln\left(-n+\sqrt{n^2+N^2}\right)-2.\ln(N)}\right\}$

I(4)c: $\frac{3.e_1.e_2.V_n.v.k.N}{2.d^2.c^2}\cdot\int\limits_0^n\int\limits_0^n\frac{n^2.dn}{\left(n^2+N^2\right)^{5/2}} = \frac{e_1.e_2.V_n.v.k}{2.d^2.c^2.N}\cdot\int\limits_0^n\frac{n^3.dn}{\left(n^2+N^2\right)^{3/2}}$

$$= \frac{e_1.e_2.V_n.v.k}{2.d^2.c^2.N}\cdot\left[\sqrt{n^2+N^2}\right]_0^n + \frac{e_1.e_2.V_n.v.k.N}{2.d^2.c^2}\cdot\left[\frac{1}{\sqrt{n^2+N^2}}\right]_0^n$$

$$= \frac{\mathbf{e_1.e_2.V_n.v.k}}{\mathbf{d^2.c^2}}\cdot\left\{\frac{\mathbf{1}}{\mathbf{N}}\cdot\sqrt{\mathbf{n^2+N^2}}-\mathbf{1}+\frac{\mathbf{N}}{\sqrt{\mathbf{n^2+N^2}}}-\mathbf{1}\right\},\quad \textbf{for}\int_0^n\int_0^n \textbf{as well as for}\int_{-n}^0\int_{-n}^0$$

IIa: $$-\frac{e_1.e_2.V_n.v.k.N^3}{2.d^2.c^2}\cdot\int_0^n\int_0^n\frac{dn}{\left(n^2+N^2\right)^{5/2}} = -\frac{e_1.e_2.V_n.v.k}{6.d^2.c^2.N}\cdot\left\{3.\int_0^n\frac{n.dn}{\left(n^2+N^2\right)^{1/2}}-\int_0^n\frac{n^3.dn}{\left(n^2+N^2\right)^{3/2}}\right\}$$

$$= -\frac{e_1.e_2.V_n.v.k}{2.d^2.c^2.N}\cdot\left[\sqrt{n^2+N^2}\right]_0^n + \frac{e_1.e_2.V_n.v.k}{6.d^2.c^2.N}\cdot\left[\sqrt{n^2+N^2}\right]_0^n + \frac{e_1.e_2.V_n.v.k.N}{6.d^2.c^2}\cdot\left[\frac{1}{\sqrt{n^2+N^2}}\right]_0^n$$

$$= -\frac{\mathbf{e_1.e_2.V_n.v.k}}{\mathbf{d^2.c^2.N}}\cdot\sqrt{\mathbf{n^2+N^2}} + \frac{\mathbf{e_1.e_2.V_n.v.k}}{\mathbf{d^2.c^2}} + \frac{\mathbf{e_1.e_2.V_n.v.k}}{\mathbf{3.d^2.c^2.N}}\cdot\sqrt{\mathbf{n^2+N^2}} - \frac{\mathbf{e_1.e_2.V_n.v.k}}{\mathbf{3.d^2.c^2}}$$

$$+\frac{\mathbf{e_1.e_2.V_n.v.k.N}}{\mathbf{3.d^2.c^2}}\cdot\frac{\mathbf{1}}{\sqrt{\mathbf{n^2+N^2}}} - \frac{\mathbf{e_1.e_2.V_n.v.k}}{\mathbf{3.d^2.c^2}},\quad \textbf{for}\int_0^n\int_0^n \textbf{as well for}\int_{-n}^0\int_{-n}^0$$

IIb: $$\frac{e_1.e_2.V_n^2.k.N^2}{2.d^2.c^2}\cdot\int_0^n\int_0^n\frac{n.dn}{\left(n^2+N^2\right)^{5/2}} = -\frac{e_1.e_2.V_n^2.k.N^2}{6.d^2.c^2}\cdot\int_0^n\frac{dn}{\left(n^2+N^2\right)^{3/2}} = -\frac{e_1.e_2.V_n^2.k}{6.d^2.c^2}\cdot\left[\frac{n}{\left(n^2+N^2\right)^{1/2}}\right]_0^n$$

$$= \frac{e_1.e_2.V_n^2.k}{6.d^2.c^2}\cdot\frac{n}{\left(n^2+N^2\right)^{1/2}}\cdot(+1-1).\quad \sum_{\mathbf{IIb}}=\mathbf{0}$$

IIc: $$-\frac{e_1.e_2.v^2.k.N}{2.d^2.c^2}\cdot\int_0^n\int_0^n\frac{n.dn}{\left(n^2+N^2\right)^{5/2}}\ :\ \sum_{\mathbf{IIc}}=\mathbf{0}$$, see IIb.

IId: $$\frac{e_1.e_2.V_n.v.k.N}{2.d^2.c^2}\cdot\int_0^n\int_0^n\frac{n^2.dn}{\left(n^2+N^2\right)^{5/2}} = \frac{e_1.e_2.V_n.v.k}{6.d^2.c^2.N}\cdot\int_0^n\frac{n^3.dn}{\left(n^2+N^2\right)^{3/2}}$$

$$= \frac{e_1.e_2.V_n.v.k}{6.d^2.c^2.N}\cdot\left[\sqrt{n^2+N^2}\right]_0^n + \frac{e_1.e_2.V_n.v.k.N}{6.d^2.c^2}\cdot\left[\frac{1}{\sqrt{n^2+N^2}}\right]_0^n$$

$$= \frac{\mathbf{e_1.e_2.V_n.v.k}}{\mathbf{3.d^2.c^2.N}}\cdot\sqrt{\mathbf{n^2+N^2}} - \frac{\mathbf{e_1.e_2.V_n.v.k}}{\mathbf{3.d^2.c^2}} + \frac{\mathbf{e_1.e_2.V_n.v.k.N}}{\mathbf{3.d^2.c^2}}\cdot\frac{\mathbf{1}}{\sqrt{\mathbf{n^2+N^2}}} - \frac{\mathbf{e_1.e_2.V_n.v.k}}{\mathbf{3.d^2.c^2}}$$

$$\textbf{for}\int_0^n\int_0^n \textbf{as well as for}\int_{-n}^0\int_{-n}^0$$

Suppose: $\ln[n+(n^2+N^2)^{1/2}] = A$; $\ln[-n+(n^2+N^2)^{1/2}] = A_1$; $(n^2+N^2) = B$.
Then the sum of all terms (**bold**) can be calculated:

$$-\frac{e_1.e_2.k}{d^2}\cdot A \qquad -\frac{e_1.e_2.k}{d^2}\cdot A_1 \qquad +2\cdot\frac{e_1.e_2.k}{d^2}\cdot\ln(N)$$

$$+\frac{1}{4}\cdot\frac{e_1.e_2.V_R{}^2.k}{d^2.c^2}\cdot A \qquad +\frac{1}{4}\cdot\frac{e_1.e_2.V_R{}^2.k}{d^2.c^2}\cdot A_1 \qquad -\frac{1}{2}\cdot\frac{e_1.e_2.V_R{}^2.k}{d^2.c^2}\cdot\ln(N)$$

$$-\frac{1}{2}\cdot\frac{e_1.e_2.V_n{}^2.k}{d^2.c^2}\cdot A \qquad -\frac{1}{2}\cdot\frac{e_1.e_2.V_n{}^2.k}{d^2.c^2}\cdot A_1 \qquad +1\cdot\frac{e_1.e_2.V_n{}^2.k}{d^2.c^2}\cdot\ln(N)$$

$$-\frac{3}{4}\cdot\frac{e_1.e_2.v^2.k}{d^2.c^2}\cdot A \qquad -\frac{3}{4}\cdot\frac{e_1.e_2.v^2.k}{d^2.c^2}\cdot A_1 \qquad +\frac{6}{4}\cdot\frac{e_1.e_2.v^2.k}{d^2.c^2}\cdot\ln(N)$$

$\sum_{total} = \mathbf{0}$, because: $\ln\left(n+\sqrt{n^2+N^2}\right)+\ln\left(-n+\sqrt{n^2+N^2}\right)=2.\ln(N)$

$$-\frac{2}{3}\cdot\frac{e_1.e_2.V_n.v.k}{d^2.c^2.N}\cdot(B)^{1/2} \quad -\frac{2}{3}\cdot\frac{e_1.e_2.V_n.v.k.N}{d^2.c^2}\cdot(B)^{-1/2} \quad +\frac{4}{3}\cdot\frac{e_1.e_2.V_n.v.k}{d^2.c^2}$$

$$+1\cdot\frac{e_1.e_2.V_n.v.k}{d^2.c^2.N}\cdot(B)^{1/2} \quad +1\cdot\frac{e_1.e_2.V_n.v.k.N}{d^2.c^2}\cdot(B)^{-1/2} \quad -1\cdot\frac{e_1.e_2.V_n.v.k}{d^2.c^2}$$

$$-1\cdot\frac{e_1.e_2.V_n.v.k}{d^2.c^2.N}\cdot(B)^{1/2} \quad +\frac{1}{3}\cdot\frac{e_1.e_2.V_n.v.k.N}{d^2.c^2}\cdot(B)^{-1/2} \quad -\frac{2}{3}\cdot\frac{e_1.e_2.V_n.v.k}{d^2.c^2}$$

$$+\frac{2}{3}\cdot\frac{e_1.e_2.V_n.v.k}{d^2.c^2.N}\cdot(B)^{1/2} \quad +\frac{1}{3}\cdot\frac{e_1.e_2.V_n.v.k.N}{d^2.c^2}\cdot(B)^{-1/2} \quad -\frac{2}{3}\cdot\frac{e_1.e_2.V_n.v.k}{d^2.c^2}$$

\+

$$0 \qquad +1\cdot\frac{e_1.e_2.V_n.v.k.N}{d^2.c^2}\cdot(B)^{-1/2} \quad -\frac{e_1.e_2.V_n.v.k}{d^2.c^2}$$

$$\mathbf{f_{br(n/n)}} = -\frac{\mathbf{e_1.e_2.V_n.v.k}}{\mathbf{d^2.c^2}}\cdot\left\{\mathbf{1}-\frac{\mathbf{N}}{\sqrt{\mathbf{n^2+N^2}}}\right\}$$

NUMERIC SOLUTIONS (RE-TYPED); **I.V.T.** (page 43).
(INTEGRAL REPLACED BY A SUM OVER 1000 POINTS)

```
10 A=0.25:N=1000
20 FOR  I= 1 TO 1000
30 C=(COS(;PI*I/N))^2
40 C1=(SIN(;PI*I/N))^2
50 C2=C*C1
60 X=A*C/((A^2+4*C1)^1.5)
70 Y=Y+X
80 X1=A/((A^2+4C1)^1.5)
90 Y1=Y1+X1
100 X2=A*C2/((A^2+4*C)^2.5)
110 Y2=Y2+X2
120 NEXT I
130 Z1=;PI*4E-7*Y*;PI/N
140 Z2=;PI*2E-7*Y1*;PI/N
150 Z3=;PI*24E-7*Y2*;PI/N
160 PRINT A,Z1,Z2,Z3,Z1-Z2+Z3
```

$$F_R = 4.\pi.i_I.i_{II}.10^{-7}\int_0^{\pi}\frac{q.\cos^2\beta.d\beta}{\left(q^2+4.\sin^2\beta\right)^{3/2}}$$

$$-2.\pi.i_I.i_{II}.10^{-7}\int_0^{\pi}\frac{q.d\beta}{\left(q^2+4.\sin^2\beta\right)^{3/2}}$$

$$+24.\pi.i_I.i_{II}.10^{-7}\int_0^{\pi}\frac{q.\sin^2\beta.\cos^2\beta.d\beta}{\left(q^2+4.\sin^2\beta\right)^{5/2}}$$

q	Z1	Z2	Z3	Z(1-2+3)
0.25	4.91062721E-06	-2.55117545E-06	2.35945176E-06	**4.71890333E-06**
0.50	2.33778664E-06	-1.30309943E-06	1.03463719E-06	**2.06937438E-06**
1.00	1.02147670E-06	-6.62293870E-07	3.59182836E-07	**7.18365672E-07**
2.00	3.76406855E-07	-3.00037632E-07	7.63692232E-08	**1.52738446E-07**
4.00	1.13335676E-07	-1.04601954E-07	8.73372272E-09	**1.74674454E-08**
100.00	1.97362486E-10	-1.97332892E-10	2.95940152E-14	**5.91880216E-14**

NUMERIC SOLUTIONS (RE-TYPED); PROF.WYDER; K.U.N. (page 43).
(INTEGRAL REPLACED BY A SUM OVER 1000 POINTS)

$$F = -2\pi.i_I.i_{II}.\left(\frac{\alpha_0}{4\pi}\right)\cdot\int_0^{2\pi}\frac{\alpha.\cos\phi.d\phi}{\left(2+\alpha^2-2.\cos\phi\right)^{3/2}}$$

```
100  PRINT "ALPHA" ; \ INPUT A
200  N=1000
210  Y=0
1000 FOR I=1 TO N
1100 C=COS(2*PI*I/N)
1200  X=A*C/((2+A^2-2*C)^1.5)
1300  Y=Y+X
1500  NEXT I
2000  Z=PI*-2.00000E-07*Y*2*PI/N
3000  PRINT A,Z
4000  GO TO 100
9999  END
```

ALPHA (=q)	**FORCE**
0.25	**47.1891 E-7**
0.50	**20.6937 E-7**
1.00	**7.18365 E-7**
2.00	**1.52738 E-7**
4.00	**0.17467 E-7**
100.00	**5.919 E-14**

Data, mentioned in § 9.8.2, pages 104-107.

Text of a page of site *http://teyssier.home.cern.ch/teyssier/node5.html*

(this site does not exist anymore).

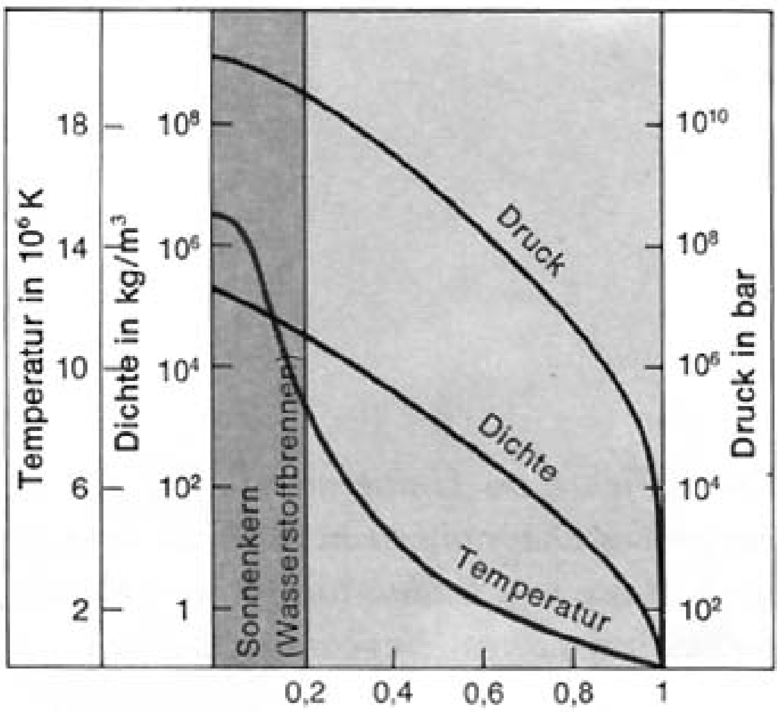
La masse du boson de Higgs

La limite supérieure indirecte sur la masse du Higgs est actuellement de 220 GeV; elle est basée sur des mesures de précision des observables électrofaibles qui sont sensibles aux corrections radiatives impliquant des échanges de bosons de Higgs virtuels.

Par contre, la limite inférieure, est déterminée par les recherches directe au LEP. Elle est actuellement, dans l'expérience L3, de 95.3 GeV, avec un degré de confiance de 95 %.

Daniel Francois Teyssier
2001-01-23

Figure from "Materie in Raum und Zeit" (page 166)



Data, obtained from the figure

Part of radius	Density ρ Kg/m^3	Pressure Influence by 0.06846 P_A	Temperature T × 10^6 K	Energy MeV MeV	log(MeV)
0.04	2.00×10^5	1.26×10^7	15.1×10^6	2.246×10^8	8.351
0.08	1.26×10^5	1.00×10^7	14.9×10^6	1.115×10^8	8.047
0.12	1.00×10^5	7.94×10^6	14.1×10^6	6.651×10^7	7.823
0.16	7.94×10^4	6.31×10^6	13.9×10^6	4.137×10^7	7.617
0.20	5.01×10^4	5.01×10^6	13.0×10^6	1.938×10^7	7.287
0.24	2.51×10^4	2.51×10^6	12.0×10^6	4.491×10^6	6.652
0.28	1.58×10^4	6.31×10^5	10.6×10^6	6.278×10^5	5.798
0.32	1.26×10^4	2.51×10^5	10.0×10^6	1.879×10^5	5.274
0.36	1.00×10^4	7.94×10^4	8.0×10^6	3.773×10^4	4.577
0.40	6.31×10^3	5.01×10^4	7.5×10^6	1.408×10^4	4.149
0.44	3.98×10^3	3.16×10^4	6.6×10^6	4.931×10^3	3.693
0.48	2.51×10^3	1.58×10^4	6.0×10^6	1.414×10^3	3.150
0.52	1.58×10^3	1.00×10^4	5.1×10^6	4.787×10^2	2.680
0.56	1.00×10^3	6.31×10^3	4.5×10^6	1.687×10^2	2.227
0.60	6.31×10^2	2.51×10^3	4.0×10^6	3.763×10	1.576
0.64	3.98×10^2	1.26×10^3	3.4×10^6	1.013×10	1.006
0.68	2.51×10^2	6.31×10^2	3.0×10^6	2.823	0.451
0.72	1.58×10^2	3.98×10^2	2.4×10^6	0.897	-0.047
				Average:	**4.22**

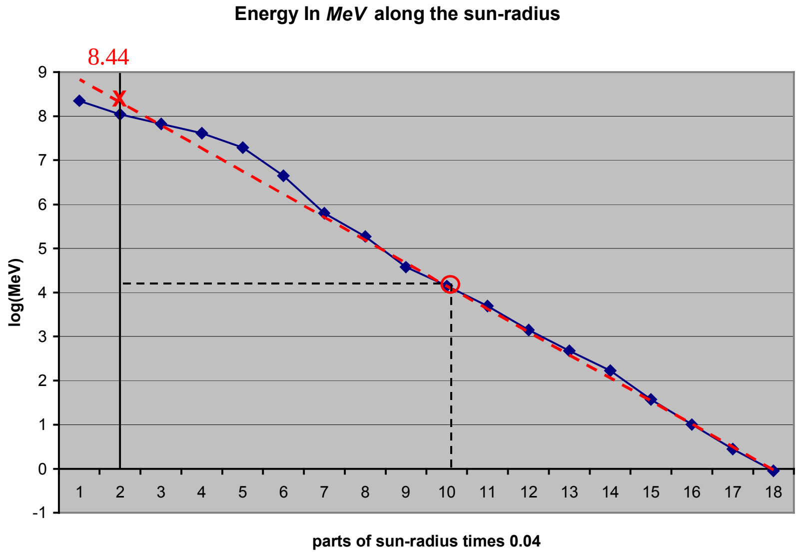
Using the Universal Gas-constant : R = 5.188×10^{16} *MeV.Kmol^{-1}.T^{-1}*, and the atom-weight of m_p = 1.6726×10^{-27} *kg* , it is necessary to account for changing the units.
1 Kmol(H)-gas at *1 P_A* and *20^0 C* takes a constant volume (22.4 *m^3*). Changing the units of temperature from ^{0}C to T, a **factor of reduction** must be introduced with the value 20 / (272.16 + 20) **= 0.06846** . Because P.V/T is a constant, P.V must also be decreased by that factor. At V = 1 m^3, only P (*N.m^{-2}*) can take over the reduction:

E = R × m_p × T × ρ × 0.06846 × P_A *MeV*,

(or E = T × ρ × P_A × 5.94054 × 10^{-12} *MeV*)

(E in *MeV*, T in *Kelvin* and density *ρ in kg.m^{-3}*).

It is obvious that near the centre of the sun the values of log (MeV) show some irregularities of the straight line. Trying to correct that, a choice has been made for the straight-line outcome at about 0.1 of the radius. Thus idealized:
log 8.44 to 4.22 over the radii 0.1 - 0.4 and log 4.22 to 0 over the radii 0.4 - 0.7.
Log (8.44) → 2.754×10^8 *MeV* ; thus the mean value is $(2.754\times10^8)^{1/2}$= 1.66×10^4 *MeV.*



Solution of the point of time of the average gravitational effect.

The point of time of that average can be found from the equation on page 110.
If *A'* is put on 10^x *m* and the mean effect on 50, that point of time can be solved from the equation:

$$\frac{1+1.592\times10^{-19+x}}{1+1.588\times10^{-21+x}}=50$$

1+ 1.592 × 10^{-19+x} = 50 + 7.94 10^{-20+x} ; 1+ 15.92 × 10^{-20+x} = 50 + 7.94 × 10^{-20+x}
15.92 × 10^{-20+x} - 7.94 × 10^{-20+x} = 49 ; 7.98 × 10^{-20+x} = 49 ;
10^{-20+x} = 6.14 = $10^{0.788}$; -20 + x = 0.788 ; x = 20.788 ;
A' = $10^{20.788}$ *m* = 6.1376 × 10^{20} *m* = 6.487 × 10^4 *lj* , or 0.65 × 10^5 *light years*.
0.65×10^5/(1.5×10^{10}) = 4.33×10^{-6}, giving:
3.79 ×10^{75} × 4.33 ×10^{-6} ≈ 1.6 ×10^{70} *J/T$_U$*.
This total power may be compared to E_U = - E_{GU} = 1.7 ×10^{70} *J/T_U* (p. 156-170).