

4. Forces between electric currents in conductors

In the preceding chapters it has been demonstrated that an elementary charge, which is moving relatively with respect to another *ec*, acts at any moment on that *ec* as being elongated along the track, covered in the period of *pp*-convergence. From the model, in which an *ec* moves uniformly along a straight line, two equations were deduced by means of the energon hypothesis, describing the axial and transversal forces respectively (§ 3.2):

$$f_{g1} \approx f_0 \cdot \{1 + (v^2/4c^2) + (v^2/4c^2) \cdot \cos^2 \alpha\}$$

$$f_{g2} \approx f_0 \cdot (v^2/2c^2) \cdot \sin \alpha \cdot \cos \alpha$$

Only in the case of *ec*'s, flowing through conductors, uniform and straight-on motion can be expected. The influences between current conducting wires are very well known, so that these systems are very suitable to test the two equations and by doing so, to make the energon hypothesis more believable. In the following paragraphs the test will be carried out. It was necessary to introduce the conception of single-row conductors, which means hypothetical conductors consisting of one row of positively charged metallic ions along which easily moving electrons can flow. In a later stage, the single-row conductors are bundled into real conductors and, after that, bend into a circular shape. It will be proved that the two equations are very well able to explain the diverse influences and to make a quantitative description of it as well.

4.1. Straight, single-row conductors being in rest

The most simple conductor, which is imaginable, may be the single-row conductor, consisting of a straight row of mutual fixed, positively charged metallic ions and, connected to that, a single row of movable electrons. In **figure 25 a** the symbol *d* indicates the distances between the centres of the composing atoms and the distances between the migrating electrons. The combination of an ion and a moving electron is called a **magnetom**.

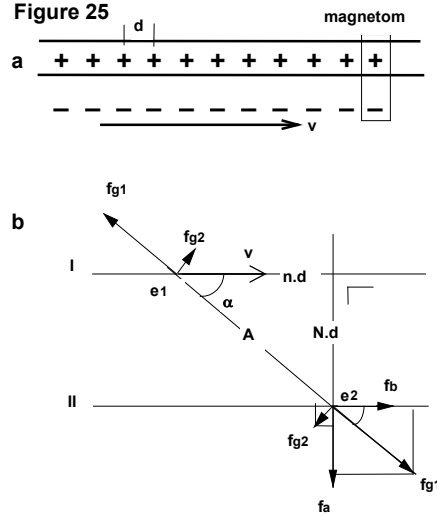
In **figure 25 b** the symbols *I* and *II* indicate two parallel single-row conductors, and *e*₁ and *e*₂ indicate two arbitrary electrons on those conductors, flowing with velocity *v*.

The vectors *f*_{g1} and *f*_{g2}, according to the above mentioned equations, can be resolved into components along the conductors and perpendicular to it, and subsequently be added two by two, resulting in the equations *f*_b and *f*_a, respectively:

$$f_a = f_{g1} \cdot \sin\alpha + f_{g2} \cdot \cos\alpha = f_o \cdot \sin\alpha \cdot \{1 + (v^2/4c^2) + (3v^2/4c^2 \cdot \cos^2\alpha)\}$$

$$f_b = f_{g1} \cdot \cos\alpha + f_{g2} \cdot \sin\alpha = f_o \cdot \cos\alpha \cdot \{1 + (3v^2/4c^2) - (v^2/4c^2 \cdot \cos^2\alpha)\}$$

In order to be able to describe the resulting force between all similar charges on *I* and on *II*, the charges on *I* must be imagined to be numbered with respect to the line through e_2 , perpendicular to the conductors, i.e. $+n$ on one side and $-n$ on the other side. Hence, the distance of the charges to the perpendicular will be $\pm n \cdot d$, if the distance between the (point-like) charges is called d . If the distance between the two conductors is expressed as $N \cdot d$, the integrated force of *I* on e_2 can be calculated as follows.



The force between e_1 and e_2 can be described by:

$$f_0 = \frac{e_1 \cdot e_2 \cdot k}{A^2} = \frac{e_1 \cdot e_2 \cdot k}{(n^2 + N^2) \cdot d^2}; k = (4 \cdot \pi \cdot \epsilon_0)^{-1}$$

The sine and cosine of the angle α can be expressed as

$$\sin\alpha = \frac{N}{(n^2 + N^2)^{1/2}}; \cos\alpha = \frac{n}{(n^2 + N^2)^{1/2}}$$

These equations, introduced to those of f_a and f_b gives:

$$f_a = \frac{e_1 \cdot e_2 \cdot k}{d^2} \cdot \frac{N}{(n^2 + N^2)^{3/2}} \cdot \left\{ 1 + \frac{v^2}{4c^2} + \frac{3v^2}{4c^2} \cdot \frac{n^2}{(n^2 + N^2)} \right\}$$

$$f_b = \frac{e_1 \cdot e_2 \cdot k}{d^2} \cdot \frac{n}{(n^2 + N^2)^{3/2}} \cdot \left\{ 1 + \frac{3v^2}{4c^2} - \frac{v^2}{4c^2} \cdot \frac{n^2}{(n^2 + N^2)} \right\}$$

Integration of both equations over n allows to express the total of the forces:

$$f_{a(1/n)} = \int_{-n}^{+n} f_a \cdot dn = \frac{2 \cdot e_1 \cdot e_2 \cdot k}{d^2} \cdot \frac{n}{N \cdot (n^2 + N^2)^{1/2}} + \frac{e_1 \cdot e_2 \cdot k \cdot v^2}{2 \cdot d^2 \cdot c^2 \cdot N} \left\{ \left(\frac{n^2}{n^2 + N^2} \right)^{1/2} + \left(\frac{n^2}{n^2 + N^2} \right)^{3/2} \right\}$$

$$f_{b(1/n)} = \int_{-n}^{+n} f_b \cdot dn = 0$$

If $f_{a(1/n)}$ is expressed as $f_{a(1/n)} = f_{a0} + (v^2/2c^2) \cdot R$, hence:

$$R = \frac{e_1 \cdot e_2 \cdot k}{d^2 \cdot N} \cdot \left\{ \left(\frac{n^2}{n^2 + N^2} \right)^{1/2} + \left(\frac{n^2}{n^2 + N^2} \right)^{3/2} \right\}$$

the following possibilities of force exertion can be distinguished:

$$e_2^+ / n \cdot e_1^+ : f_{a(1/n)1} = +f_{a0}$$

$$e_2^+ / n \cdot e_1^- : f_{a(1/n)2} = -f_{a0} - (v_I^2 / 2c^2) \cdot R$$

$$e_2^- / n \cdot e_1^+ : f_{a(1/n)3} = -f_{a0} - (v_{II}^2 / 2c^2) \cdot R$$

$$e_2^- / n \cdot e_1^- : f_{a(1/n)4} = +f_{a0} + (v_I + v_{II})^2 \cdot R / (2c^2)$$

$$\frac{+}{f_{ar(1/n)} = \frac{v_I \cdot v_{II}}{c^2} \cdot R = \frac{e^2 \cdot k}{d^2} \cdot \frac{v_I \cdot v_{II}}{c^2} \cdot \frac{1}{N} \cdot \left\{ \left(\frac{n^2}{n^2 + N^2} \right)^{1/2} + \left(\frac{n^2}{n^2 + N^2} \right)^{3/2} \right\}}$$

This equation describes the perpendicular force between a magnetom on the middle of a single-row conductor and all magnetoms on another single-row conductor in front of, and parallel to, the first one, if an electric current flows through both of them.

The velocities v_I and v_{II} are equally signed, if the sense of electronic movement on both conductors is equal (thus absolutely reversed). The velocities are oppositely signed with oppositely sensed electronic movement (absolutely equal).

An equal sign results in a positive or recoiling force, and an opposite sign in a negative or attracting force.

If the single-row conductors are infinite ($n_1 = \infty$), the force becomes:

$$f_{ar(1/\infty)} = 2 \cdot \frac{v_I \cdot v_{II}}{c^2} \cdot \frac{e^2 \cdot k}{d^2 \cdot N}$$

The total force for n magnetoms on II measures:

$$f_{ar(n/\infty)} = 2 \cdot \frac{v_I \cdot v_{II}}{c^2} \cdot \frac{e^2 \cdot k \cdot n}{d^2 \cdot N}$$

If $n = N$, the force becomes:

$$f_{ar(n/\infty)} = 2 \cdot \frac{v_I \cdot v_{II}}{c^2} \cdot \frac{e^2 \cdot k}{d^2}$$

This equation describes the perpendicular force on each piece of infinite and parallel conductors, if these pieces have lengths of $n \cdot d$ and distances $n \cdot d$.

4.2. Straight, normal conductors being in rest

For application to normal conductors, the last equation has to be recalculated to

conductors with a circular profile with diameter $2r$. The number of single-row conductors within that profile measure $\pi \cdot r^2 d^{-2}$, so that the equation has to be multiplied by the square of that number:

$$F_{a(n/\infty)} = 2 \cdot \pi^2 \cdot \frac{v_I \cdot v_{II}}{c^2} \cdot \frac{e^2 \cdot k \cdot r^4}{d^6}$$

On its turn, this equation can be transformed into an equation for currents in *ampere*, in stead of electronic velocities. To this purpose the next data must be used:

- i = current in *ampere* (A)
- Z = quantity of silver, precipitated in one second by 1A
- Az = atomic weight of silver
- Ak = atomic weight of copper
- Dk = density of copper in atoms per volume
- NA = number of Avogadro
- Sgk = specific gravity of copper

The electronic velocity (v) and the density of copper (Dk) can be defined by:

$$v = \frac{i \cdot Z \cdot NA}{Az \cdot Dk \cdot \pi \cdot r^2}; \quad Dk = \frac{Sgk \cdot NA}{Ak} = d^{-3}$$

from which may be deduced:

$$v^2 \cdot d^{-6} = \frac{i^2 \cdot Z^2 \cdot NA^2}{Az^2 \cdot \pi^2 \cdot r^4}$$

This equation, applied to that for $F_{a(n/\infty)}$ gives:

$$F_{a(n/\infty)} = 2 \cdot \pi^2 \cdot \frac{i_I \cdot i_{II} \cdot Z^2 \cdot NA^2}{Az^2 \cdot \pi^2 \cdot r^4} \cdot \frac{e^2 \cdot k \cdot r^4}{c^2} = 2 \cdot i_I \cdot i_{II} \cdot k \cdot \left(\frac{e \cdot Z \cdot NA}{Az \cdot c} \right)^2$$

which describes the perpendicular force on each piece of two infinite and parallel conductors in vacuum, if those pieces have lengths of m and distances of m , on condition that the diameters may be neglected with respect to the distance. The use of k depends on the chosen system of units.

The circumstances, which are chosen, are like those of the definition of the ampere. Thus, substitution of the right data must result into a force of $\pm 2.00000 \times 10^{-2}$ dyne, or $\pm 2.00000 \times 10^{-7}$ Newton.

These data are:

$$\begin{aligned}
 i_I &= i_{II} = 1 \text{ A} & Z &= 1.1180 \times 10^{-6} \text{ kg} \cdot \text{s}^{-1} \\
 e &= 1.6021892 \times 10^{-19} \text{ C (SI-system)} & Az &= 107.87 \\
 &= 4.80324 \times 10^{-10} \text{ esu (1 esu} = 1\text{C} \cdot 10\text{c)} & NA &= 6.022045 \times 10^{26} \text{ ec's kmol}^{-1} \\
 k^{-1} &= 4\pi\epsilon_0 = 10^7 \times c^{-2} \text{ C} \cdot \text{V}^{-1} \cdot \text{m}^{-1} \text{ (SI-system)} & c &= 2.997925 \times 10^8 \text{ m} \cdot \text{s}^{-1}
 \end{aligned}$$

According to the cm/g/s-system the following calculation is valid (c in cm.s⁻¹)

$$F_{a(n/\infty)} = 2 \times \left(\frac{4.80324 \times 10^{-10} \times 1.118 \times 10^{-3} \times 6.022045 \times 10^{23}}{107.87 \times 2.997925 \times 10^{10}} \right)^2$$

$$\text{Thus: } F_{a(n/\infty)} = 2 \times i_I \times i_{II} \times [(e \cdot Z \cdot NA) / (Az \cdot c)]^2 = \pm 2.00000 \times 10^{-2} \text{ dyne (cm/g/s)}$$

The calculation in the SI-system is somewhat different:

$$F_{a(n/\infty)} = \frac{2}{10^7 \times c^{-2}} \times \left(\frac{1.6021892 \times 10^{-19} \times 1.118 \times 10^{-6} \times 6.022045 \times 10^{26}}{c \times 107.87} \right)^2$$

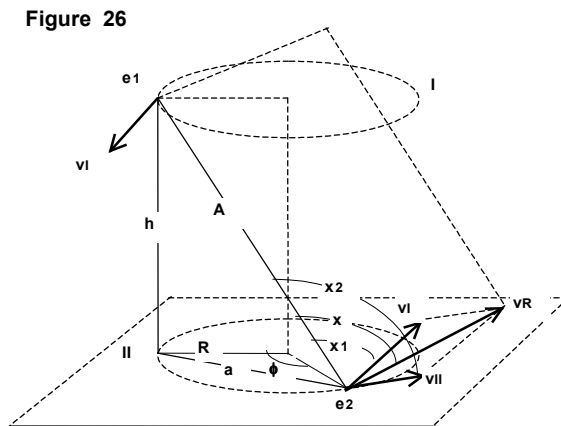
$$\text{Thus: } F_{a(n/\infty)} = 2 \times i_I \times i_{II} \cdot k \times [(e \cdot Z \cdot NA) / (Az \cdot c)]^2 = \pm 2.00000 \times 10^{-7} \text{ N (SI)}$$

As can be seen, the equations derived from the energon hypothesis are able to describe the forces, which are working under the circumstances at which the ampere is defined.

4.3. Circular conductors, perpendicular to a joint axis and being in rest

As circular shaped conductors are commonly used, this shape is chosen here to get a second proof of the validity of the two equations, derived from the energon hypothesis. **Figure 26** depicts the velocities of two arbitrary ec's, moving in equal circular conductors, situated perpendicularly on a joint axis: the velocity of an ec on

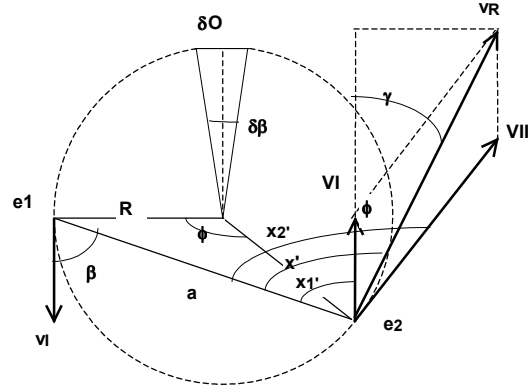
conductor I by means of vector v_I , and the velocity of an ec on conductor II, by means of vector v_{II} . The resultant of vector v_I and the relative vector v_I on II is depicted by v_R .



The angles between the connecting line A and the vectors v_I , v_{II} and v_R are indicated by x_1 , x_2 and x , respectively.

The perpendicular projection of figure 26 on the plane through conductor II is given in **figure 27**, whereas the relation between the real angles x and the projection of these angles x' is shown in **figure 28 b**. The vectors of force, working between both ec's in a plane through connecting line A and the concerning vector of velocity v_R , are depicted in **figure**

Figure 27



28 a. It must be remarked, that v_I and v_{II} should be signed oppositely in the situation as pictured, because the velocities are oppositely sensed (absolute equally directed).

From figure 27 can be deduced:

$$v_R^2 = v_I^2 + v_{II}^2 + v_I \cdot v_{II} \cdot \cos \phi ;$$

$$\cos \gamma = -(v_I/v_R) - (v_{II}/v_R) \cdot \cos \phi ;$$

$$\sin \gamma = (v_{II}/v_R) \cdot \sin \phi ;$$

$$x' = \frac{1}{2} \cdot \phi + \gamma ; \quad x_1' = \frac{1}{2} \cdot \phi ; \quad x_2' = 180^\circ - \frac{1}{2} \cdot \phi$$

Figure 28 b shows:

$$a \cdot \cos(x') = A \cdot \cos(x); \quad a \cdot \cos(x_1') = A \cdot \cos(x_1); \quad a \cdot \cos(x_2') = A \cdot \cos(x_2).$$

From figure 28 a can be found that the components of force, perpendicular to A and projected on h , measure:

$$f_{2a} = f_2 \cdot (h/A) \cdot (\cos x / \sin x), \text{ because}$$

$$f_{2a} = f_2 \cdot \sin(x-90^\circ) \cdot \sin \phi = f_2 \sin(x-90^\circ) \cdot (h/A) = f_2 \cdot \sin(x-90^\circ) \cdot h / (A \cdot \sin x)$$

According to the following equations:

$$\mathbf{f}_{g1} \approx \mathbf{f}_0 \cdot \{1 + (v^2/4c^2) + (v^2/4c^2) \cdot \cos^2 \chi\} \text{ and}$$

$$\mathbf{f}_{g2} \approx \mathbf{f}_0 \cdot (v^2/2c^2) \cdot \sin \alpha \cdot \cos \alpha,$$

the direct force components on the ec's are:

$$f_1 = \frac{e^2 \cdot k}{A^2} \cdot \left(1 + \frac{v_R^2}{4c^2} + \frac{v_R^2}{4c^2} \cdot \cos^2 \chi \right)$$

$$f_2 = \frac{e^2 \cdot k}{A^2} \cdot \frac{v_R^2}{2c^2} \cdot \sin \chi \cdot \cos \chi$$

Thus, the components along h are given by:

$$f_{1a} = \frac{e^2 \cdot k}{A^2} \cdot \frac{h}{A} \cdot \left(1 + \frac{v_R^2}{4c^2} + \frac{v_R^2}{4c^2} \cdot \cos^2 \chi \right)$$

$$f_{2a} = \frac{e^2 \cdot k}{A^2} \cdot \frac{h}{A} \cdot \left(\frac{v_R^2}{2c^2} \cdot \cos^2 \chi \right)$$

Addition of these equations gives:

$$f_{ap} = \frac{e^2 \cdot k \cdot h}{A^3} \cdot \left(1 + \frac{v_R^2}{4c^2} + \frac{3v_R^2}{4c^2} \cdot \cos^2 \chi \right)$$

This equation has to be applied to the interaction between:

e^-/e^- , with velocity v_R and angle x ($= f_{ap}$)

e^-/e^+ , with velocity v_I and angle x_1 ($= f_{aq}$)

e^+/e^- , with velocity v_{II} and angle x_2 ($= f_{ar}$)

e^+/e^+ , without velocities ($= f_{as}$)

ad f_{ap}

$\cos x = (a/A) \cdot \cos(x') = (a/A) \cdot \cos(\frac{1}{2} \cdot \phi + \gamma)$, and, if $\frac{1}{2} \cdot \phi$ is put on β :

$$\cos x = (a/A) \cdot (\cos \beta \cdot \cos \gamma - \sin \beta \cdot \sin \gamma);$$

$$\cos \beta \cdot \cos \gamma = \cos \beta \{ -(v_I/v_R) - (v_{II}/v_R) \cos \phi \}$$

$$\cos \beta \cdot \cos \gamma = -(v_I/v_R) \cos \beta - (v_{II}/v_R) \cos \beta \cdot \cos \gamma$$

$$= -(v_I/v_R) \cos \beta - (v_{II}/v_R) \cos \beta (2 \cos^2 \beta - 1)$$

$$= -(v_I/v_R) \cos \beta + (v_{II}/v_R) \cos \beta - (2v_{II}/v_R) \cos^3 \beta$$

$\sin \beta \cdot \sin \gamma = \sin \beta (v_{II} \cdot \sin \phi / v_R) = (v_{II}/v_R) \sin \beta \cdot 2 \sin \beta \cdot \cos \beta$, thus

$$\cos x = a/A \{ -(v_I/v_R) \cos \beta + (v_{II}/v_R) \cos \beta - (2v_{II}/v_R) \cos^3 \beta - (2v_{II}/v_R) \sin^2 \beta \cdot \cos \beta \}$$

$$= a/A \{ -(v_I - v_{II}) \cos \beta / v_R - (2v_{II}/v_R) \cos \beta (\cos^2 \beta + \sin^2 \beta) \}$$

$\cos x = -a/A \{ (v_I + v_{II}) \cos \beta / v_R \}$; $\cos^2 x = (a/A)^2 \{ (v_I + v_{II})^2 \cos^2 \beta / v_R^2 \}$, thus:

$$f_{ap} = e^2 \cdot h / A^3 \{ 1 + v_R^2 / 4c^2 + (3a^2 / 4A^2) \cdot (v_I + v_{II})^2 \cos^2 \beta / c^2 \};$$

because $v_R^2 = v_I^2 + v_{II}^2 + 2v_I \cdot v_{II} \cdot \cos \beta$

$$= v_I^2 + v_{II}^2 + 2v_I \cdot v_{II} (2 \cos^2 \beta - 1) = (v_I - v_{II})^2 + 4v_I \cdot v_{II} \cos^2 \beta$$

$$f_{ap} = e^2 \cdot h / A^3 \{ 1 + (v_I - v_{II})^2 / 4c^2 + v_I \cdot v_{II} \cos^2 \beta / c^2 + (3a^2 / 4A^2) (v_I + v_{II})^2 \cos^2 \beta / c^2 \}$$

ad f_{aq}

$$f_{aq} = -e^2 \cdot h / A^3 (1 + v_I^2 / 4c^2 + 3v_I^2 \cdot \cos^2 x_1 / 4c^2); ; A \cdot \cos x_1 = a \cdot \cos \beta; \cos x_1 = a \cdot \cos \beta / A$$

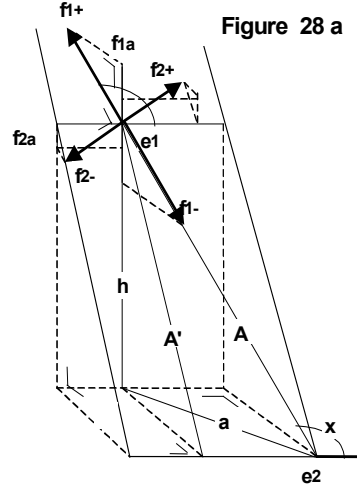
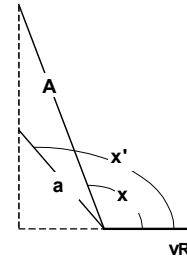


Figure 28 a

Figure 28 b



$$f_{aq} = e^2 \cdot h/A^3 \{-1 - v_{II}^2/4c^2 - (3a^2/4A^2) \cdot (v_I^2/c^2) \cos^2 \beta\}$$

ad f_{ar}

$$f_{ar} = -e^2 h/A^3 \{1 + v_{II}^2/4c^2 + 3v_{II}^2 \cdot \cos^2 x_2/4c^2\}; \cos x_2 = a \cdot \cos(x_2')/A = -a \cdot \cos \beta/A$$

$$f_{ar} = e^2 \cdot h/A^3 \{-1 - v_{II}^2/4c^2 - (3a^2/4c^2) \cdot (v_{II}^2/c^2) \cos^2 \beta\}$$

ad f_{as}

$$f_{as} = e^2 \cdot h/A^3$$

Addition of the four forces ($f_a = f_{ap} + f_{aq} + f_{ar} + f_{as}$) leads to the equation:

$$f_a = \frac{e^2 \cdot k \cdot h}{A^3} \cdot \left(-\frac{v_r \cdot v_{II}}{2c^2} + \frac{v_r \cdot v_{II}}{c^2} \cdot \cos^2 \beta + \frac{3a^2}{2A^2} \cdot \frac{v_r \cdot v_{II}}{c^2} \cdot \cos^2 \beta \right)$$

This equation describes the force between two magnetoms on the circular conductors, perpendicular to the planes through the conductors.

From the figures can also be seen:

$$a^2 = 4R^2 \sin^2 \beta; A^2 = h^2 + 4R^2 \sin^2 \beta = R^2 \cdot (q^2 + 4 \cdot \sin^2 \beta), \text{ if } h = q \cdot R.$$

The factor ($e^2 \cdot h/A^3$) can be replaced by:

$$\frac{e^2 \cdot k \cdot q}{R^2 \cdot (q^2 + 4 \cdot \sin^2 \beta)^{3/2}} = \frac{e^2 \cdot k \cdot q}{R^2 \cdot Q^{3/2}}$$

if ($q^2 + 4 \cdot \sin^2 \beta$) is indicated by Q ,

In order to find the total force between the two conductors, the circumferences O have to be multiplied by δO : $2\pi R/d \times 2R \cdot \delta\beta/d = 4\pi R^2 \cdot \delta\beta/d^2$ (where d is the distance between two magnetoms and $2R \cdot \sin(\delta\beta)/d = 2R \cdot \delta\beta/d$).

The product ought to be used as a factor for the equation of f_a that subsequently has to be integrated over β , altering from π to 0 :

$$\frac{4\pi R^2 \cdot d\beta}{d^2} \times \frac{e^2 \cdot k \cdot q}{R^2 \cdot Q^{3/2}} = \frac{4\pi \cdot e^2 \cdot k \cdot q}{d^2 \cdot Q^{3/2}}$$

$$f_R = \pi \cdot q \cdot \left(\frac{v_r \cdot v_{II}}{c^2} \cdot \frac{e^2 \cdot k}{d^2} \right) \cdot \int_0^\pi \left(-\frac{2 \cdot d\beta}{Q^{3/2}} + \frac{4 \cdot \cos^2 \beta \cdot d\beta}{Q^{3/2}} + \frac{24 \cdot \sin^2 \beta \cdot \cos^2 \beta \cdot d\beta}{Q^{5/2}} \right)$$

As has been found before, the force: $2(e^2 \cdot k/d^2) \cdot (v_I \cdot v_{II})/c^2$ for single-row conductors, can be transformed into a force for normal conductors with circular profile and diameter $2r$: $2\pi^2 \cdot (e^2 \cdot r^4 \cdot k/d^6) \cdot (v_I \cdot v_{II}/c^2)$, see the equations for $f_{ar(n/-)}$ and for $F_{a(n/-)}$, or into:

$$2 \cdot i_I \cdot i_{II} \cdot k \cdot \left(\frac{e \cdot Z \cdot NA}{AZ \cdot c} \right)^2 = 2 \cdot i_I \cdot i_{II} \cdot 10^{-7} \text{ N}, \text{ for currents in } \textit{ampere}.$$

With the application to normal circular conductors and currents in *ampere*, the above equation becomes:

$$F_R = \pi \cdot i_I \cdot i_{II} \cdot 10^{-7} \cdot \left(-2 \cdot \int_0^\pi \frac{q \cdot d\beta}{Q^{3/2}} + 4 \cdot \int_0^\pi \frac{q \cdot \cos^2 \beta \cdot d\beta}{Q^{3/2}} + 24 \cdot \int_0^\pi \frac{q \cdot \sin^2 \beta \cdot \cos^2 \beta \cdot d\beta}{Q^{5/2}} \right)$$

$$\text{with } Q = q^2 + 4 \cdot \sin^2 \beta$$

In the case that the two conductors are equal, with diameters which may be neglected with respect to the distance, and with currents of +1A, the force between the two conductors have been calculated by means of a computer for the distances 0.25- 0.50- 1.00- 2.00- 4.00- and $100.0 \times q$ ($q = h/R$) :

$$F(0.25) = 4.718904 \times 10^{-6} \text{ N } (4.71891 \times 10^{-6} \text{ N})$$

$$F(0.50) = 2.069374 \times 10^{-6} \text{ N } (2.06937 \times 10^{-6} \text{ N})$$

$$F(1.00) = 7.183657 \times 10^{-7} \text{ N } (7.18365 \times 10^{-7} \text{ N})$$

$$F(2.00) = 1.527384 \times 10^{-7} \text{ N } (1.52738 \times 10^{-7} \text{ N})$$

$$F(4.00) = 1.746745 \times 10^{-8} \text{ N } (1.7467 \times 10^{-8} \text{ N})$$

$$F(100) = 5.91880 \times 10^{-14} \text{ N } (5.919 \times 10^{-14} \text{ N})$$

The values between brackets were found from the equation, officially used for problems like this, by prof. Wyder at the Kath. University of Nijmegen:

$$F = -2\pi \cdot i_I \cdot i_{II} \cdot \left(\frac{\mu_0}{4\pi} \right) \cdot \int_0^{2\pi} \frac{q \cdot \cos \phi \cdot d\phi}{(2 + q^2 - 2 \cdot \cos \phi)^{3/2}}$$

in which $\mu_0 = 4\pi \cdot 10^{-7} \text{ H} \cdot \text{m}^{-1}$ and $\phi = 2\beta$.

See Chapter 14, page XVii.

As our equation has been derived from the fundamental thesis about force-exertion by converging energons with velocities of $c \pm 1/2c$, leading to $\Delta t = A \cdot \sqrt{2}/c$, the close agreement between both equations proves the validity of the thesis.

4.4 Effect of perpendicular motion of parallel conductors on currents

If a constant electric current is flowing through a straight conductor, there is no influence on a second conductor without a current and being parallel to the first one. However, if this second conductor is moved towards the other a counter directed current starts to flow through it, resulting in a recoiling force. With an opposite motion, thus moving away, a current starts to flow into the same direction as that in the first conductor (opposite sense of ec-motion) causing an attracting force.

In **figure 29** the above described situations have been pictured. On the conductor with a flowing current, the positively charged ions only have a velocity V_n , if that conductor starts to move with that velocity towards a second conductor, parallel to the first one, whereas the electrons move with V_n as well with v , perpendicular to V_n . At the starting position, both kinds of charges are in rest on conductor II. In the case of single-row conductors and observation during a very short period of time (Δt), the forces on the concerning charges may be described by:

$$f_{g1} = e^2 \cdot k / A^2 \cdot \{1 + V_{R2}^2 / 4c^2 + (V_R^2 / 4c^2) \cdot \cos^2 \alpha\}, \text{ and}$$

$$f_{g2} = e^2 \cdot k / A^2 \cdot (V_R^2 / 2c^2) \cdot \sin \alpha \cdot \cos \alpha ,$$

according to the equations of the energy hypothesis, in which the average distance between the conductors is expressed by A , the resultant of v and V_n by V_R , the angle between A and V_R by α , the angle between V_R and v by β .

If the two forces are resolved into components along the conductors (b) and perpendicular to it (a), one will find:

$$f_{g1b} = f_{g1} \cdot \cos(\alpha + \beta)$$

$$f_{g2b} = f_{g2} \cdot \sin(\alpha + \beta). \text{ together } f_b , \text{ and}$$

$$f_{g1a} = f_{g1} \cdot \sin(\alpha + \beta)$$

$$f_{g2a} = f_{g2} \cdot \cos(\alpha + \beta), \text{ together } f_a.$$

The sine and cosine of the angle $(\alpha + \beta)$ can be described by:

$$\sin(\alpha + \beta) = N(n^2 + N^2)^{-1/2} \text{ and } \cos(\alpha + \beta) = n(n^2 + N^2)^{-1/2} ,$$

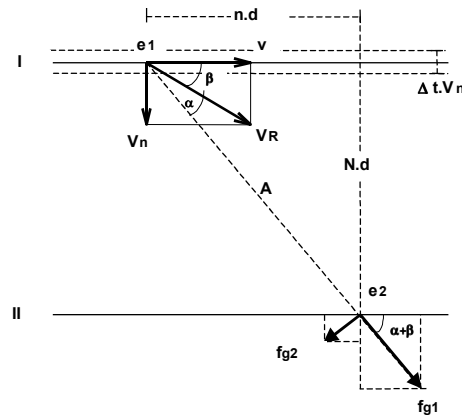
so that we will find:

$$f_b = f_{g1} \cdot n(n^2 + N^2)^{-1/2} + f_{g2} \cdot N(n^2 + N^2)^{-1/2}$$

A charge in II, on a line perpendicular to the middle of I, is influenced by all the similar charges on I. A force along the conductor results. This may be described by:

$$f_{br(1/n)} = \int_{-n}^n f_b \cdot dn = \int_{-n}^n \frac{f_{g1} \cdot n \cdot dn}{\sqrt{n^2 + N^2}} + \int_{-n}^n \frac{f_{g2} \cdot N \cdot dn}{\sqrt{n^2 + N^2}} , \text{ thus:}$$

Figure 29



$$f_{br(1/n)} = -\frac{e_I \cdot e_{II} \cdot k}{d^2} \cdot \frac{V_n \cdot v}{c^2} \cdot \frac{1}{N} \cdot \left\{ \left(\frac{n^2}{n^2 + N^2} \right)^{1/2} - \frac{2}{3} \cdot \left(\frac{n^2}{n^2 + N^2} \right)^{3/2} \right\}$$

This force originates exclusively from the moving ec's on *I*, because the positively charged particles do not move along the conductor.

The sign of V_n is negative at approach and positive at increase of the distance. A negative V_n causes a positive force on the electron along the conductor, which means a force into opposite direction with respect to the primary current. A positive V_n , however, causes a negative force on the electron on *II*, pointing into the same direction as the primary current does.

The force between two equal pieces of single-row conductors, being parallel and right in front of each other, can be found by double integration of the following function:

$$f_b = f_{g1} \cdot n(n^2 + N^2)^{-1/2} + f_{g2} \cdot N(n^2 + N^2)^{-1/2}, \text{ according to:}$$

$$f_{br(n/n)} = \int_{-n}^n \left(\int_{-n}^n f_b \cdot dn \right) \cdot f_b \cdot dn = -\frac{e_I \cdot e_{II} \cdot k}{d^2} \cdot \frac{V_n \cdot v}{c^2} \left(1 - \sqrt{\frac{N^2}{n^2 + N^2}} \right)$$

This equation shows that a positive V_n (fleeing) causes a negative force on all electrons on *II*, thus causing an equally directed current with an attraction between the conductors, whereas a negative V_n (approaching) causes a positive, counter directed secondary current, with a recoiling force between the conductors.

It may be remarked that the force can be described by:

$$f_{br(\sim/n)} = -(e_I \cdot e_{II} \cdot k/d^2) \cdot (V_n \cdot v/c^2), \text{ if } n \gg N,$$

whereas the force becomes zero, if $N \gg n$.

The conclusion of these deductions is, that the equations from the energon hypothesis appear to be valid for the description of the electromagnetic effects between moving conductors.

4.5. The effect of current variation in parallel conductors

In the case of changing currents, the deduction of the equations, used thus far, needs some adaptation.

If the Δt -track ($S_{\Delta t}$) of a uniformly moving ec is situated perpendicular to the average distance A with respect to another ec (symmetrical situation), the average force f_{g2} is eliminated. However, it can be shown that a force along the conductors remains, if the velocity of the ec is increasing uniformly from v_0 to v_t during the period of time Δt (period of pp-convergence). The effects are pictured in **figure 30 a** and **figure 30 b**.

In the case of a uniformly altering current the Δt -track measures:

$$S_{\Delta t} = v_0 \cdot \Delta t + a \cdot \Delta t^2 / 2,$$

in which the acceleration of the ec's is indicated by a .

If A is much larger than $S_{\Delta t}$, the maximal angle β can be described by

$$\tan \beta \approx \sin \beta \approx (v_0 \cdot \Delta t + \frac{1}{2} \cdot a \cdot \Delta t^2) / 2A \approx \beta,$$

$$\text{in which } \Delta t = A \cdot \sqrt{2} / c.$$

The component of force along the conductor is:

$f_2 = f \cdot \sin \beta$, where:

$$f = e^2 \cdot k \cdot D \cdot \cos^2 \beta / A^2 = e^2 \cdot k \cdot D \cdot (1 - \sin^2 \beta) / A^2,$$

$$D \approx 1 + v^2 / 2c^2 \text{ and } v = v_0 + a \cdot t,$$

thus $f_2 = e^2 \cdot k \cdot D \cdot (1 - \sin^2 \beta) \sin \beta / A^2$,

$$\text{or } f_2 = e^2 \cdot k \cdot D \cdot \sin \beta / A^2 - e^2 \cdot k \cdot D \cdot \sin^3 \beta / A^2.$$

As $\sin \beta \approx v_0 \cdot \Delta t / 2A + a \cdot \Delta t^2 / 4A$

$$\approx v_0 \cdot \sqrt{2} / 2c + a \cdot A / 2c^2 \text{ and}$$

$D \approx 1 + (v_0 + a \cdot t)^2 / 2c^2$, the force f_2 for β -max. becomes:

$$f_2 = \frac{e^2 \cdot k}{A^2} \cdot \left\{ 1 + \frac{(v_0 + a \cdot t)^2}{2c^2} \right\} \times \left\{ \frac{v_0 \cdot \sqrt{2}}{2c} + \frac{a \cdot A}{2c^2} \right\} - \frac{e^2 \cdot k}{A^2} \cdot \left\{ 1 + \frac{(v_0 + a \cdot t)^2}{2c^2} \right\} \times \left\{ \frac{v_0 \cdot \sqrt{2}}{2c} + \frac{a \cdot A}{2c^2} \right\}^3$$

The first term can partly be ignored, because c occurs to the power -3 or -4 in the two terms of the outcome:

$$v_0 \cdot \frac{\sqrt{2}}{2c} + \frac{a \cdot A}{2c^2} + \left| \frac{v_0 \cdot \sqrt{2} \cdot (v_0 + a \cdot t)^2}{4c^3} + \frac{a \cdot A \cdot (v_0 + a \cdot t)^2}{4c^4} \right| = \frac{v_0 \cdot \sqrt{2}}{2c} + \frac{a \cdot A}{2c^2}$$

The second term can be ignored as a whole, because c occurs to the power -4 or less in all terms of the outcome.

Thus, the force f_2 for β -max. can be described by:

$$f_2 \approx (e^2 \cdot k / A^2) \cdot \{ (v_0 \cdot \sqrt{2} / 2c) + (a \cdot A / 2c^2 \dots) \}, \text{ or, if } \Delta t \text{ is re substituted:}$$

$$f_2 \approx (e^2 \cdot k / A^2) \cdot \{ (v_0 \cdot \Delta t / 2A) + (a \cdot \Delta t^2 / 4A) \dots \}, \text{ or, for smaller periods:}$$

$$f_2 \approx (e^2 \cdot k / A^2) \cdot \{ (v_0 \cdot t / 2A) + (a \cdot t^2 / 4A) \}.$$

The average force over the period Δt can be found by integration of this equation over Δt (from $\frac{1}{2}\Delta t$ to $-\frac{1}{2}\Delta t$), and dividing the outcome by Δt :

$$f_{g2} = \frac{e^2 \cdot k}{A^2 \cdot \Delta t} \cdot \int_{-\Delta t/2}^{\Delta t/2} \frac{v_0 \cdot t \cdot dt}{2A} + \frac{e^2 \cdot k}{A^2 \cdot \Delta t} \cdot \int_{-\Delta t/2}^{\Delta t/2} \frac{a \cdot t^2 \cdot dt}{4A}$$

The first term becomes zero:

Figure 30 a

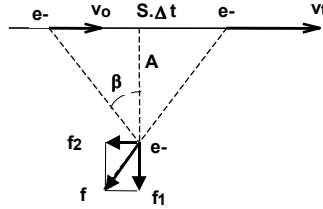
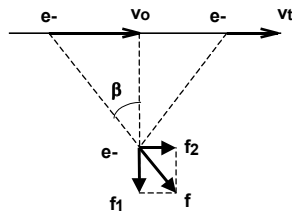


Figure 30 b



$$\frac{e^2 \cdot k \cdot v_0}{2A^3 \cdot \Delta t} \left[\frac{t^2}{2} \right]_{-\Delta t/2}^{\Delta t/2} = \frac{e^2 \cdot k \cdot v_0}{2A^3 \cdot \Delta t} \left(\frac{\Delta t^2}{8} - \frac{\Delta t^2}{8} \right) = 0$$

The second term does not vanish:

$$\frac{e^2 \cdot k \cdot a}{4A^3 \cdot \Delta t} \cdot \left[\frac{t^3}{3} \right]_{-\Delta t/2}^{\Delta t/2} = \frac{e^2 \cdot k \cdot a}{4A^3 \cdot \Delta t} \cdot \left(\frac{\Delta t^3}{24} + \frac{\Delta t^3}{24} \right) = \frac{e^2 \cdot k \cdot a \cdot \Delta t^2}{48 \cdot A^3}$$

As $\Delta t = A \cdot \sqrt{2}/c$, the force along the conductor becomes:

$$\mathbf{f}_{g2} \approx (\mathbf{e}^2 \cdot \mathbf{k} \cdot \mathbf{a}) / (24 \cdot \mathbf{A} \cdot \mathbf{c}^2) \mathbf{N}$$

From this equation can be seen that, at positive acceleration (increase of velocity), a positive force parallel to the conductor is exerted, pointing into a direction opposite to that of the primary current, and at negative acceleration (decrease of velocity) a negative force is exerted, pointing into a direction equal to that of the primary current. A positive charge in the place of the negative charge in figure 30 should experience counter directed forces from the primary current.

In the case of a double system of parallel conductors, these effects cause an opposite secondary current, accompanied by repulsion, or, an identical secondary current with attraction, respectively.

This outcome is in accordance to the experimental findings and shows that the Δt -idea, used in the energon hypothesis, is indispensable for the explanation of the relation between relatively moving elementary charges.